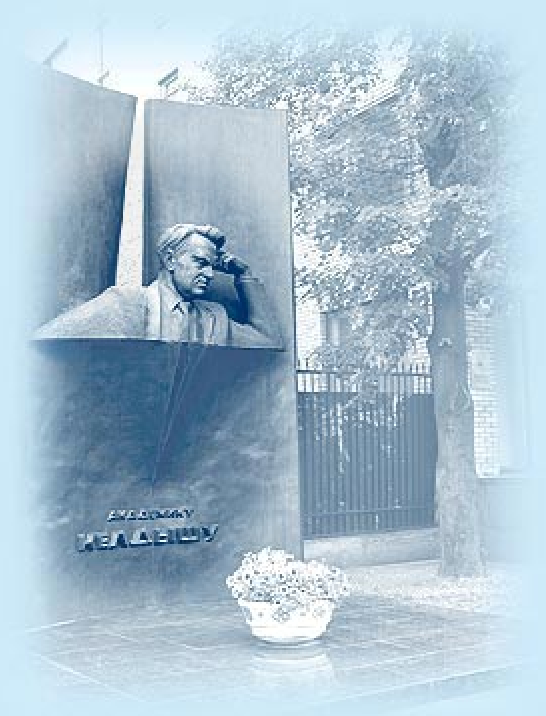

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Supersymmetric polaron

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Supersymmetric polaron

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Лахно В.Д.

Суперсимметричный полярон

Построена суперсимметричная теория полярона. Суперпартнером такой квазичастицы является заряженный фонон. Теория применима к различным типам поляронов, которые в каждом случае имеют своих суперпартнеров. Указаны способы экспериментальной проверки теории.

Ключевые слова: суперпартнер, заряженный фонон, супералгебра Ли, рамановское рассеяние, протон

Lakhno V.D.

The supersymmetric polaron

A polaron sypersymmetry theory is developed. The superpartner of this quasiparticle is a charged phonon. The theory can be applied to different polarons which have their superpartners in each case. A possible experimental verification of the theory is discussed.

Key words: superpartner, charged phonon, Lie superalgebra, Raman scattering, proton

Dedicated to memory of N.M. Plakida

1. Introduction

The theory of elementary particles assumes the possibility of exact supersymmetry at high particle energies.

At low energies, supersymmetry breaks down and superpartner particles turn out to have different masses and spectral characteristics [1] – [3]. In recent experiments at the hadron collider, however, superpartner particles were not detected (at least in the energy range accessible to the hadron collider). This fact called into question the supersymmetric theory of elementary particles.

Regardless of the existence of supersymmetry of elementary particles, its mathematical apparatus can be used to describe the properties of elementary quasiparticles in the theory of condensed systems.

In this paper, we discuss the conclusions which follow from the exact supersymmetry of translation-invariant (TI) polaron quasiparticles, whose properties underlie the explanation of the mechanism of high-temperature superconductivity (HTSC) [4, 5].

Section 2 considers the Hamiltonian of a free polaron, which is expressed in terms of creation and annihilation operators of a polaron which obey the fermion permutation relations.

Section 3 deals with the same Hamiltonian expressed in terms of creation and annihilation operators of a TI polaron, which obey the bosonic permutation relations.

This contradiction is resolved in Section 4, where we analyze a transformation which mixes bosons and fermions and combines fermions and bosons into one supermultiplet. Thus, it has been shown that a polaron and TI polarons are superpartners.

Section 5 discusses the possibility of experimental verification of the theory and its applications of the theory to various condensed media.

2. Fermionic polaron

The polaron theory is the simplest example of a non-relativistic quantum theory of a fermionic particle interacting with a bosonic field.

The Hamiltonian of such a system has the form:

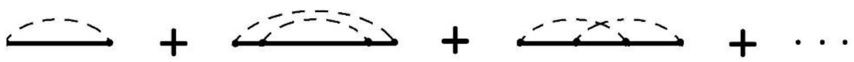
$$H = \frac{\hat{p}^2}{2m} + \sum_k \hbar\omega(k) a_k^+ a_k + \sum_k V_k \left[a_k e^{ikr} + a_k^+ e^{-ikr}\right], \quad (1)$$

where r is the electron radius-vector, $\hat{p}$ is its momentum, m is the electron effective mass; a_k^+, a_k are operators of creation and annihilation of the bosonic (phonon) field quanta with energy $\hbar\omega(k)$, V_k is the matrix element of the interaction of an electron with a bosonic field.

A polaron as a quasiparticle is an electron surrounded by a cloud of bosons, whose Green's function has the form [6], [7]:

$$G(\vec{p}, E) = \left\{E - \varepsilon_{\vec{p}} - M(\vec{p}, E)\right\}^{-1}, \quad (2)$$

where the mass operator $M(\vec{p}, E)$ is determined by the diagrammatic series:

$M(\vec{p}, E) =$  (3)

where the solid and dashed lines correspond to the free electron and bosonic Green's functions, $\varepsilon_{\vec{p}} = {p^2}/{2m}$, $E(\vec{p}) = \varepsilon_p + M\left(\vec{p}, E(\vec{p})\right)$ is the polaron energy.

Thus, a polaron, like an electron, is a fermion whose Hamiltonian can be written as:

$$H = \sum_p E(\vec{p}) c_p^+ c_p, \quad (4)$$

where c_p^+, c_p has the meaning of the fermionic operators of creation and annihilation of a polaron.

Using the idea of a free polaron (4) as a fermionic quasiparticle, we can consider its interaction with other quasiparticles (for example, scattering of an optical polaron by optical phonons [8], [9]). We note that the spin indices are not involved in Hamiltonian (4), since this does not matter in the case of an isolated quasiparticle.

3. Bosonic polaron

The above procedure leads to the idea of a polaron as a fermion, since it actually excludes the bosonic components from consideration.

When studying Hamiltonian (1), we could proceed differently and carry out relevant canonical transformations. In fact, the characteristics and spectral properties, as is known, are not changed by such transformations.

For this purpose, let us use the Heisenberg transformation, which excludes electronic coordinates from Hamiltonian (1) [10]:

$$S_1 = exp\{i \textstyle\sum_k \vec{k} a_k^+ a_k\}\vec{r} \ , \qquad (5)$$

$$S_1^{-1} a_k S_k = a_k e^{-ikr} \ , \qquad S_1^{-1} a_k^+ S_1 = a_k^+ e^{ikr} \ .$$

With the use of (5) the transformed Hamiltonian (1) takes the form:

$$\widetilde{H} = S_1^{-1} H S_1 \ , \qquad (6)$$

$$\widetilde{H} = \frac{1}{2m}\left(\textstyle\sum_k \hbar\vec{k} a_k^+ a_k\right)^2 + \textstyle\sum_k V_k (a_k + a_k^+) + \textstyle\sum_k \hbar\omega(k) a_k^+ a_k \ ,$$

and does not contain electronic coordinates.

As is shown in [10], the application of the Lee-Low-Pines canonical transformations to Hamiltonian (6), such that:

$$S_2 = exp\{\sum_k f_k (a_k^+ - a_k)\} , \qquad (7)$$
$$S_2^{-1} a_k S_2 = a_k + f_k , \qquad S_2^{-1} a_k^+ S_2 = a_k^+ + f_k ,$$

where f_k are real numbers, as well as the canonical squeezed transformation:

$$S_3 = Cexp\left\{\frac{1}{2}\sum_{k,k'} a_k^+ A_{kk'} a_{k'}^+\right\} , \qquad (8)$$
$$S_3^{-1} a_k S_3 = \sum_{k'} M_{1kk'} \alpha_{k'} + \sum_{k'} M_{2kk'} \alpha_{k'}^+ ,$$

where the explicit form of the matrices A and M is given in [10], leads to the following expression for the polaron energy:

$$\tilde{\tilde{\tilde{H}}} = S_3^{-1} S_2^{-1} S_1^{-1} H S_1 S_2 S_3 = \sum_k E_k \alpha_k^+ \alpha_k , \qquad (9)$$

where α_k^+, α_k are operators of a TI polaron which satisfy the bosonic permutation relations.

Hence a TI polaron, according to (9) is a boson.

Below we will show that a TI polaron can be both a boson and a fermion, representing an example of a supersymmetric particle.

4. Polaron as a supersymmetric particle

Since both the approaches presented above describe the same fermion-bosonic system, the spectrum of the fermionic Hamiltonian:

$$H = \sum_k E_k \mathrm{c}_k^+ \mathrm{c}_k , \qquad (10)$$

where the creation and annihilation c_k^+, c_k operators obey the Fermi permutation relations: coincides with the spectrum of the TI polaron Hamiltonian: $\{c_k c_{k'}^+\} = c_k c_{k'}^+ + c_{k'}^+ c_k = \delta_{kk'}$.

$$H = \sum_k E_k \alpha_k^+ \alpha_k \ , \tag{11}$$

where the operators of creation and annihilation of a TI polaron α_k^+, α_k obey the Bose permutation relations: $[\alpha_k \alpha_{k'}^+] = \alpha_k \alpha_{k'}^+ - \alpha_{k'}^+ \alpha_k = \delta_{kk'}$.

Let us introduce a supersymmetric Hamiltonian $\mathcal{H}$:

$$\mathcal{H} = \sum_k E_k (c_k^+ c_k + \alpha_k^+ \alpha_k) = \sum_k \mathcal{H}_k, \tag{12}$$
$$\mathcal{H}_k = E_k (c_k^+ c_k + \alpha_k^+ \alpha_k) \ .$$

Let us introduce the operators:

$$Q_k^+ = \sqrt{E_k} \alpha_k c_k^+, \qquad Q_k = \sqrt{E_k} \alpha_k^+ c_k, \tag{13}$$

where Q_k^+ is an operator which transforms a boson with a momentum k into a fermion with a momentum k, Q_k is an operator which transforms a fermion with a momentum k into a boson with a momentum k.

Then the operators Q_{1k}, Q_{2k}:

$$Q_{1k} = Q_k^+ + Q_k, \qquad Q_{2k} = -i(Q_k^+ - Q_k), \tag{14}$$

form a Lie superalgebra [11]:

$$\{Q_{ik}, Q_{jk}\} = 2\delta_{ij} \mathcal{H}_k \ , \tag{15}$$
$$[Q_{ik}, \mathcal{H}_k] = 0 \ .$$

Expressed in terms of the operators Q_i, Hamiltonian $\mathcal{H}$ has the form:

$$\mathcal{H} = \sum_k Q_{1k}^2 = \sum_k Q_{2k}^2 = \sum_k \{Q_k^+, Q_k\}. \qquad (16)$$

Hence the "fermionic" polaron described in Section 2 is the superpartner of the "bosonic" polaron described in Section 3.

5. Discussion

As shown above, TI polarons are an example of supersymmetric quasiparticles in the physics of condensed systems, which, unlike elementary particles, have an exact supersymmetry.

The supersymmetry of TI polarons has found interesting applications in condensed matter physics. Thus, the presence of supersymmetry made it possible to explain the coexistence of such particles in strongly correlated fermionic systems with ordinary fermions, in particular, to explain the nontrivial behavior of the magnetoresistance of cuprate superconductors [12].

The exact supersymmetry of TI polarons makes it difficult to study them experimentally. In the case of a weak electron-bosonic coupling, the properties of a TI polaron differ only slightly from the properties of a band electron or its superpartner - bosonic excitation (when its spin does not matter).

In the case of strong coupling the presence of TI polarons as fermions can be detected in experiments on their capture by crystal impurities [10].

The occurrence of bosonic properties of a TI polaron can be revealed when studying the Raman spectra (combination scattering). Scattering of light by TI polarons, as well as by free bosons, will contain a doublet of satellites separated by a frequency $\pm\omega(k)$ from the fundamental frequency of the scattered light. The contribution of TI polarons into this doublet can be detected from a change in their concentration in a magnetic field. According to [12], this contribution will be proportional to the concentration of TI polarons $n_H = n(k_B T/\mu_B H)th(\mu_B H/k_B T)$, where T is the temperature, H is the external magnetic field, and n is the concentration of TI polarons in the absence of a magnetic field. The contribution of ordinary bosons (if they are not magnetic excitations) will not be affected by an external magnetic field.

Supersymmetry implies doubling the number of known elementary quasiparticles due to the availability of superpartners.

We can say that in the case of polarons (optical or acoustic), these will be charged (optical or acoustic) phonons, that is, phonon TI polarons. In the case of plasmorons (electrons interacting with plasmons [13]), these will be charged plasmons, and so on.

As was noted in [4]-[5] nonrelativistic theory of nucleon interaction with meson field on the scale of nuclear force radius has the same structure as polaron theory. Thus the proton superpartner will be $\pi+$ meson, antiproton π^- meson, neutron (antineutron) π^0 meson. This is evident from hadronic resonance:

$$p + \gamma = n + \pi,$$

where the role of proton excitation spectrum is played by π mesons [14].

The availability of supersymmetry proton or neutron partner in nuclei can be detected in experiments on inelastic scattering of $\boldsymbol{\gamma}$ - quanta in nuclei [15], by revealing the pion satellites in scattered quanta analogously to what was mentioned above for the case of TI-polarons.

Superpartner particles can combine to form compound particles. For example, TI polarons can combine to form TI bipolarons. Such composite particles, as in the theory of elementary particles, have no superpartners-quasiparticles.

6. Afterword

This preprint is dedicated to a remarkable scientist and man Nikolai Maksimilyanovich Plakida. When I was at the Joint Institute for Nuclear Research (JINR) in Dubna, I asked Nikolai Maksimilyanovich, which of their scientists was in charge of superconductivity then. He told me that he was in charge [1)]. Then I gave him my manuscript on the TI-polaron theory of superconductivity and asked him to be a reviewer. Some time later, having read the manuscript, he told me that he could not be my reviewer, because he did not understand why such a fermionic particle as a polaron is described in the TI-polaron theory by bosonic creation and annihilation operators. My answer then, based on the concept of bound phonons, did not convince him, since charged phonons in solid state physics were not known to anyone.

This article provides an answer to Nikolai Maksimilyanovich's question, which could possibly convince him. Unfortunately, he is no longer with us and we will never know about it.

1) Being the author of a fundamental monograph on superconductivity published by Springer (Plakida N.M. (2010), High Temperature Cuprate Superconductors: Experiment, Theory and Applications, Hidelberg, Germany) Nikolai Maksimilyanovich had every right to such an answer.

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Contents