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It is pointed out that the translation-invariant bipolaron theory of superconductivity can explain the possibility of a metal-superconductor or insulator-superconductor quantum phase transition. The coherence length of a gas of translationally invariant bipolarons is calculated. It is shown that in very dirty superconductors an insulating state characterized by the presence of a coherence peak can be formed. It is concluded that it is impossible for a charge density wave with a non-zero wave vector to propagate in the nodal direction. It is shown that dirty superconductors can be used to create room-temperature superconductivity.

Key words: quantum computers, stochastic potential, Bose condensate, nanostructures, superconducting gap

1. Introduction

Dirty superconductors (SC) have been studied for more than three decades, but a full-fledged theory that would explain all their oddities is still missing [1], [2] (see also reviews [3] - [4]). This, in particular, prevents the creation of quantum computers, which require maximum isolation from the outside world and other electronic devices based on similar materials.

The aim of this paper is to draw attention to the fact that a number of aspects of the behavior of dirty superconductors can directly follow from the microscopic translationally invariant (TI) bipolaron theory of superconductivity [5] and can be used to create room-temperature superconductors. The peculiarity of bipolaron TI superconductivity is that the role of Cooper pairs there is played by TI bipolarons – delocalized bound states of electrons or holes, whose ground state energy can be lower than that of bipolarons with broken symmetry (BS bipolarons) captured by the attractive potential of the defects in the ionic crystal structure of high-temperature SCs such as vacancies, various types of impurities, island or fluctuating potential of a dirty SC. For BS polarons and bipolarons, an arbitrarily weak potential well leads to their localization in this well, while in the case of a TI polaron or bipolaron, a finite depth of this well is required for their capture by the well. In the case of individual bipolarons, quantum transitions between the TI and BS states of bipolarons are possible when the value of the attractive trap potential changes. If the value of the attractive trap potential U is small, the TI state of the bipolaron is realized, and when the value of the attractive potential exceeds a certain critical value U_c, the BS state is realized. A similar situation takes palace in the Anderson impurity model if the energy of the TI bipolaron is lower than the energy of the localized state in the stochastic potential. This conclusion is in accordance with Anderson's theorem [6], [7], based on the Bardeen-Cooper-Schrieffer (BCS) theory [8], which states that at a small level of disorder (fluctuations of the stochastic potential), the critical temperature T_c remains unchanged. When a certain threshold U_c is exceeded, the delocalized wave function of the TI bipolaron collapses into the wave function of the BS bipolaron localized at the impurity. At a temperature T less than the temperature T_c corresponding to the SC transition, and U less than U_c, delocalized TI bipolarons form a SC Bose condensate, and at U greater than U_c this Bose condensate collapses into a dielectric or poorly conducting state (bad metal) with broken symmetry of BS bipolarons localized on impurities.

This situation can occur in dirty SCs during quantum transitions of the metal-superconductor or insulator-superconductor type [1] - [4], as well as in SC films, where such a transition to a super-insulating state occurs with a decrease in its thickness [9], [10]. The opposite situation is also possible when, with a decrease in T, due to the peculiarities of the potential U, a transition from the SC to the insulating state occurs. This type of transition was probably observed in [11].

An important role in describing the properties of dirty SCs is played by the coherence length. The coherent length determines the characteristic size of the Bose condensate, at which the corresponding order parameter changes little. This size determines the properties of the entire Bose condensate and, generally speaking, is not related to the size of the particles that form the Bose condensate state. In particular, the coherence length can be a temperature-dependent quantity, while the particle size remains unchanged with a change in temperature. However, the Bose condensation effect, although manifested macroscopically, it is fundamentally a quantum phenomenon in which the wave function of an individual particle has macroscopic dimensions. If a particle has a spatial structure, then its topology can determine the topology of the entire Bose condensate. This question is of current importance, for example, when describing SC in nanostructures, the characteristic size of which may be smaller than the coherence length of a superconducting Bose condensate. The question of the magnitude of the coherent length acquires particular importance in dirty SCs, where a nonuniform distribution of the order parameter is established. Physical phenomena in which quantum coherence plays a fundamental role also include the Josephson effect and magnetic flux quantization.

2. Coherence length of a TI bipolaron Bose condensate

The complex order parameter in the macroscopic theory of SC is the wave function of the Bose condensate, the modulus of which is proportional to the size of the SC gap. Accordingly, in the bipolaron theory of superconductivity, the gap size is the phonon frequency. Note that the squared modulus of the wave function of the Bose condensate has the meaning of the density of the number of TI bipolarons in the condensate. In this case, the coherence length in the microscopic BCS theory is inversely proportional to the width of the SC gap.

In quantum statistical mechanics, the off-diagonal elements of the density matrix determine the degree of coherence between different quantum states. In the case of a Bose gas under consideration, its state is described by plane wave functions of particles with momentum k: $\varphi_k(r_i) = e^{ikr_i}/\sqrt{V}$, where r_i are the coordinates of the i-th particle, V is the volume of the system. Accordingly, the off-diagonal element of the density matrix, which is a key parameter for understanding the long-range order, is determined by the expression:

$$g(r_1, r_2) = \sum_k n_k \varphi_k^*(r_1)\varphi_k(r_2), \quad (1)$$

where $n_k = \{exp[(E_k - \mu)/T] - 1\}^{-1}$ is the Bose distribution function of gas particles. Separating the contributions of condensate with k=0 from the temperature component, we can rewrite this expression in the form:

$$g(r_1, r_2) = \frac{N_0}{V} + \frac{1}{(2\pi\hbar)^3}\int d^3k \frac{exp[ik(r_i - r_2)/\hbar]}{exp[(E_k - \mu)/T] - 1}, \quad (2)$$

where N_0 is the number of particles in the Bose condensate. The energy of the TI bipolaron E_k involved in (2), according to [5], has the form:

$$E_k = \left[\Delta_k + E_{bp} + k^2/2M\right],\ k > 0\ , \tag{3}$$

where E_{bp} is the energy of the bipolaron ground state, $\Delta_k = \omega_k$ is the phonon frequency, which has the meaning of the size of the superconducting gap:

$$\Delta_k = \omega_0, \tag{4}$$

- for an s-type gap and an optical phonon whose frequency ω_0 does not depend on k, and:

$$\Delta_k = \omega_0 + \Delta_0\left|\cos k_x a - cos k_y a\right|, \tag{5}$$

- for an s+d-type gap, where $M = 2m$, m is the effective mass of an electron (hole).

Using (3)-(4) for g_R in the case of an s-type gap we obtain:

$$g(R) = \frac{N_0}{V} + \frac{1}{l_T^3}\sum_{j=1}^{\infty} exp\left[-j\left(E_{bp} + \omega_0 - \mu\right)/T - \pi R^2/j l_T^2\right] j^{-3/2}\ , \tag{6}$$

$$l_T = \hbar\left(\frac{2\pi}{MT}\right)^{1/2}, \tag{7}$$

where $R = r_1 - r_2$. In the case of $R \gg l_T$:

$$g(R) = \frac{N_0}{V} + \frac{2}{l_T^2 (2\pi\xi R)^{1/2}} K_{1/2}\left(R/\xi\right), \tag{8}$$

where K_μ is the modified Bessel function,

$$\xi = l_T \Big/ \left(2\pi^{1/2}\sqrt{\left(E_{bp} + \omega_0 - \mu\right)/T}\right). \tag{9}$$

The quantity ξ has the meaning of the coherence length. Taking into account (7), the expression for ξ can be presented as:

$$\xi = \hbar \Big/ \sqrt{2M\left(\hbar\omega_0 + \quad E_{bp} - \mu\right)} \tag{10}$$

From (10) it follows that the coherence length of a TI bipolaron gas at $T \leq T_c$ does not depend on T. At $T \geq T_c$ the temperature dependence of the coherence length, according to (10), is determined by the temperature dependence of the chemical potential $\mu(T)$. At $T = T_c$, when $\mu = E_{bp}$ the coherence length ξ of an ideal Bose gas (IBG), corresponding to $\omega_0 = 0$, goes to infinity and remains so at $T \leq T_c$.

As distinct from IBG, the coherence length of the TI bipolaron gas is finite at $T \leq T_c$ and equal to:

$$\xi = \hbar \Big/ \sqrt{2M\hbar\omega_0} \tag{11}$$

At $T \geq T_c$, the coherence length remains finite and is determined by (10) up to the temperature of existence of the pseudogap phase T^* at which the decay of TI bipolarons occurs.

Note that for $R \gg \xi$ the quantity $g(R)$, according to (8), has the form:

$$g(R) = 1 \Big/ l_T^2 R \; exp\left(- R/\xi\right) \tag{12}$$

In the case of YBCO with $\hbar\,\omega_0 = 7{,}5 \cdot 10^{-3}$ eV, $M = 2m_0$, for the coherence length ξ at $T \leq T_c$ from (11) we obtain: $\xi = 1{,}6 \cdot 10^{-7}$cm $(= 16\text{Å})$. Accordingly, the correlation length: $l_{corr} = \hbar^2 \tilde{\varepsilon} \mathrm{X}(\eta)/me^2$ for $\tilde{\varepsilon} = 4$, $\mathrm{X}(\eta) = 7$ is equal to: $l_{corr} = 15$. Recall that in the BCS theory, the coherent length ξ is defined as the size of a Cooper pair and is equal to: $\xi = \hbar v_f/\pi\Delta$, v_f is the Fermi electron velocity, the typical value of which in low-temperature SCs is of the order of $\xi \approx 10^{-4}$ cm.

Thus, for the used parameter values in high-temperature SCs $l_{corr} \approx \xi$, although, in contrast to the BCS, ξ and l_{corr} in the bipolaron theory are determined by completely different relationships between the parameters. Taking into account that the typical values for the London penetration length are $\lambda \sim 1500$Å, from the above estimates it follows that the inequality: $\lambda > \xi/\sqrt{2}$, which determines whether a SC belongs to the second kind, is satisfied for YBCO with a large margin, in full accordance with the experiment.

In an external magnetic field, the coherence length ξ becomes field dependent: $\xi = \xi(H)$. Taking this dependence into account, according to [5], is achieved by replacing the phonon frequency ω_0 involved in (11) with $\widetilde{\omega}_0 = \omega_0(1 - H^2/H_{max}^2)$ where H_{max} is the the maximum value of the magnetic field for which the homogeneous state of the SC is maintained. Thus, in a sufficiently strong field the condition $\xi(H) > l_{corr}$ will be done and at $H \to H_c$, where H_c is the maximum magnetic field at a finite temperature, the condition $\xi(H \to H_c) > \sqrt{2}\lambda$ can be fulfilled that is, the transition of a second-order superconductor to a first-order superconductor occurs. In this case, however, the homogeneous state at $H \to H_c$ becomes unstable with respect to the formation of Abrikosov vortices and the formation of vortex matter.

In the anisotropic case, the resulting expressions should be modified using in (12) instead of R a value that has the form in the main axes: $R = \sqrt{\sum_i x_i^2 \gamma_i}$, $\gamma_i = M_i/M$, $i = 1, 2, 3$, where the index *i* numbers the principal axes, $M_i = 2m_i$. This allows us to introduce the concept of the anisotropic coherence length with the components: $\xi_i =$

$\xi/\sqrt{\gamma_i}$. Accordingly, in a magnetic field, $\widetilde{\omega}_0 = \omega_0\left(1 - \sum_i H^2/H_{i,max}^2\right)$ should be used instead of ω_0, where $H_{i,max}$ is the maximum value of the magnetic field along the *i*-th axis.

In the case of an s+d gap of type (5), the first term on the right side of (5) corresponds to the contribution of the *s*-type wave, and the second term to the contribution of the d-type wave. The following condition follows from (5), when the main contribution to integral (2) comes from the values $k \approx \sqrt{2MT}$ and $ka \ll 1$. In this case, from (2) and (5) we obtain: $\Delta_0\left|\cos k_x a - cos k_y a\right|/T \cong \Delta_0 M a^2/\hbar^2 \ll 1$. Thus, at $\omega_0 \geq \Delta_0 M a^2 T/\hbar^2$, the main contribution to the integral will be made by the *s*-type wave. In this case, the results obtained above for the case of an *s*-type condensate will remain unchanged. For example, in the case of YBCO the value of ω_0/Δ_0 is ≈ 0.15 [12], [13], and the *s*-approximation condition in this case is satisfied with high accuracy.

The above calculation of the coherence length actually applies to a gas of low-density TI bipolarons in the absence of a Fermi surface. In the case when a Fermi surface with a sharp boundary is present, the considered TI bipolaron gas will have some peculiarities. The spectrum of the TI bipolaron in this case was considered in [5], [14]. According to [5], [14], a change in the spectrum will be due to the existence of Kohn anomalies [15], leading to a renormalization of phonon frequencies near the wave vector of the charge density wave. Instead of (3) in this case, according to [5], [14], we will have:

$$E_k(P_{CDW}) = E_{bp}(\vec{P}_{CDW}) + \omega_0(\vec{P}_{CDW}, \vec{k}) + {k^2}/{2M} - {\vec{k}\vec{P}_{CDW}}/{M_{bp}}, \quad (13)$$

where $\vec{P}_{CDW}$ is the wave vector of the charge density, M_{bp} is the mass of a TI bipolaron. The use of (13) leads to the modified correlation function $\tilde{\tilde{g}}(r_1, r_2)$:

$$\tilde{\tilde{g}}(r_1, r_2) = \tilde{g}(r_1, r_2) exp\left[i\, {M}/{M_{bp}}\, \vec{P}_{CDW}(\vec{r}_1 - \vec{r}_2)/\hbar\right]. \quad (14)$$

The correlation function $\tilde{g}(r_1, r_2)$ involved in (14) differs from $g(r_1, r_2)$ by replacing the gap value Δ_k in the latter with the value $\Delta_k - {MP_{CDW}^2}/{2M_{bp}^2}$. The coherence length, determined by (9)-(11), does not change. From (14), in particular, it follows that the propagation of charge density waves with non-zero P_{CDW} in the nodal direction is impossible.

3. Discussion of the results

A large number of experimental studies which confirm the bipolaron nature of the so-called preformed pairs (that is, paired electronic states formed in the pseudogap phase before their transition to the SC state) is given in the work [16]. The existence of preformed pairs is indirectly evidenced by numerous experiments on measuring magnetoresistance in thin films [17]-[23].

The paper [24] presents the results of experimental observation of Fröhlich bipolarons on the crystalline surface of high-temperature $SrTiO_3$, obtained by vacuum annealing, using angle-resolution photoemission spectroscopy (ARPES) and studying the X-ray absorption spectrum. Combined spectroscopic experiments in an alternating electric field and experiments on neutron scattering with metal-insulator transitions in $RNiO_3$ nickelates such as $LaNiO_3$ and $NdNio_3$ in [25] were interpreted on the basis of the mechanism of formation of a bipolaron Bose condensate in them, similar to the situation in high-temperature bismuth-based superconductors. A discussion of a large number of experiments which prove the existence of bipolarons in high-temperature superconductors is also made in [5].

Direct experimental evidence of the existence of localized preformed pairs using scanning tunneling spectroscopy in disordered superconductors was obtained in [2]. The occurrence of pairs in the absence of a Bose condensate in a highly disordered sample in an insulating state was associated in [2] with the absence of coherent peaks there that are available in the same purified sample. Pairs localized on impurities can form a local Bose condensate on impurity islands which are not connected to each other and conduction can be realized through the Josephson tunneling mechanism between the islands.

Note that, according to the bipolaron theory, at a very high concentration of impurities or defects of the order of $1/\xi^3$, a state is also possible when electron pairs, even being attached by their centers to defects, form a Bose condensate, thereby forming an insulating state. A characteristic feature of such a Bose condensate will be the presence of a nonzero coherence length ξ in the insulating state and the appearance of a coherent peak in the dependence of conductance on voltage, similar to the case of a pure sample.

The described properties of TI bipolarons can be used to enhance T_c and create room-temperature superconductors. According to [5], the solution to this problem is associated with solving the problem of increasing the concentration of TI bipolarons, which, as a rule, is less than a percent of the total number of current carriers. To increase their concentration, it was proposed in [5] to use a dirty superconductor with magnetic impurities, based on the fact that, unlike electrons (holes), bipolarons have zero spin. From the above, however, it follows that ordinary non-magnetic impurities can also be used for this purpose. They can be used to separate ordinary electron gas and the TI bipolaron gas of a high-temperature superconductor dissolved in it. This problem is similar to that of isotope separation in the uranium problem of the atomic project. As with the problem of uranium enrichment, various engineering solutions are possible here. As one option, samples can be created with a non-uniform distribution of impurities in a sample. For example, one can create an increased concentration of impurities in the peripheral part of the sample and a reduced concentration in the depth of the sample (this situation is often realized in films).

The parameters of the impurity should be selected so that it may capture electrons, leaving TI bipolarons delocalized. Then, while maintaining the overall electrical neutrality of the sample, a region with an increased concentration of TI bipolarons in the depth of the sample and an increased concentration of electrons at its periphery will be formed in the sample. The deep part in this case will have a higher transition temperature T_c than in a sample with a uniform impurity distribution.

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