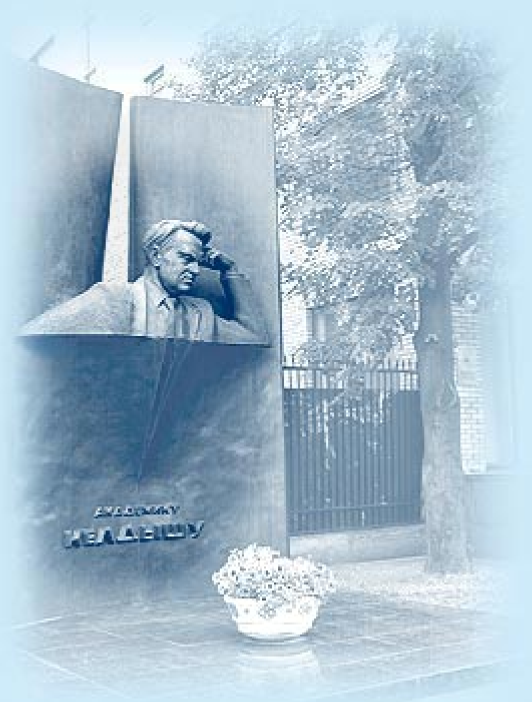

ISSN 2071-2898 (Print)
ISSN 2071-2901 (Online)

V.D. Lakhno

On the possibility of Bose condensate formation in ultracold neutron gas

Recommended form of bibliographic references: Lakhno V.D. On the possibility of Bose condensate formation in ultracold neutron gas // Keldysh Institute Preprints. 2024. No. 85. 15 p.
https://doi.org/10.20948/prepr-2024-85-e
https://library.keldysh.ru/preprint.asp?id=2024-85&lg=e

Keldysh Institute of Applied Mathematics
of Russian Academy of Sciences

Lakhno V.D.

On the possibility of Bose condensate formation in ultracold neutron gas

Moscow – 2024

V.D. Lakhno

On the possibility of Bose condensate formation in ultracold neutron gas.

It is suggested that the anomalous leakage of ultra-cold neutrons from a neutron trap may be associated with the formation of dineutrons in it. It is shown that the gas of ultracold neutrons at temperatures T less than 10^{-3} K should form a dineutron Bose condensate even in the absence of dineutrons as free stable particles. The consequences arising from the assumption of the stability of dineutrons in neutron stars are considered. The conditions for the formation of a Bose condensate from dineutrons in them and in heavy nuclei are discussed.

Key words: dineutron, superconductivity, neutron star, neutronium, nanodot, bipolaron.

1. Introduction

The search for the dineutron has been the subject of extensive literature [1-6]. The hope for the existence of such a stable particle was inspired by the existence of a stable deuteron and the presence of isotopic invariance of nucleons. Indeed, in nuclear decay reactions, the dineutron usually arises as a short-lived formation at energies of the order of the binding energy of a deuteron [1-6].

The question of the stability and existence of the dineutron at lower and ultra-low energies remains open at present. Great advances in the study of Bose condensates of ultra-cold atoms on the one hand and studies of the gas of trapped ultra-cold neutrons on the other hand, raise the question of the possibility of the formation of a neutron Bose condensate in such traps. The formation of a neutron Bose condensate is possible only if dineutrons are formed in a neutron gas, since neutrons are fermions. This possibility is supported by the presence of an anomaly in an ultra-cold neutron gas placed in a trap, when the observed amount of neutron leakage from the trap exceeds theoretical estimates by orders of magnitude [7-9]. Popular hypotheses explaining this phenomenon include such things as low heating during storage of ultracold neutrons associated with the rotation of the earth, neutron oscillations, the contribution of exotic decay processes (for example, the decay of a neutron with the formation of a neutral hydrogen atom), the transition of a neutron to dark matter, the reactor antineutrino anomaly, the existence of mirror dark matter, Zeno's quantum paradox, and others [9]. This indicates that there is currently no generally accepted explanation for the neutron anomaly.

The existence of such an anomaly could be explained by the presence of dineutrons in the ultracold gas, which, can be assumed to have a much larger capture cross-section by the nuclei of the trap walls than neutrons. If dineutrons exist, they can form a Bose condensate.

The purpose of this article is to analyze the consequences that follow from the assumption of the possibility of the existence of stable dineutrons. In this general formulation, this question is also of interest in the case of superdense matter, such as heavy atomic nuclei, neutron or boson stars.

2. Bose Condensate of Dineutrons

We will assume that the dineutron is stable at low energies.

If we assume that the concentration of dineutrons n in the trap is equal to $n = N/V$ (where N is the number of dineutrons, V is the volume of the trap) is sufficient for the formation of their Bose condensate, then $N = N_0 + N'$:

$$N = \sum_k n_k \tag{1}$$

$$N_0 = 1/(\exp(E_0 - \mu_{chem})/T - 1)$$

$$N' = \sum_{k \neq 0} 1/(\exp(E_k - \mu_{chem})/T - 1)$$

where n_k is the number of dineutrons with the wave number k, N_0 is the number of dineutrons in the ground state of the Bose condensate, N' is the number of supracondensate particles, μ_{chem} is the chemical potential of the Bose gas: $\mu_{chem} = E_{dn}$, where E_{dn} is the energy of the ground state of the dineutron. The equality $\mu_{chem} = E_{dn}$ corresponds to an infinitely large density of states of the Bose condensate near the gap $\Delta = \mu$ in the dineutron spectrum (see Appendix A):

$$E_k = [E_{dn} + \Delta_k + k^2/2M], \quad k > 0 \tag{2}$$

$$\Delta_k = \sqrt{\mu^2 + k^2},$$

where $E_0 = E_{dn}$ for $k = 0$, M is the mass of a dineutron. Here it is assumed that $\hbar = c = 1$.

Replacing the summation in (1) with integration over the wave numbers, we express the temperature of the Bose condensate of dineutrons T_c as:

$$T_c = \left(F_{3/2}(0)/F_{3/2}(\xi)\right)^{2/3} T_c(0) \tag{3}$$

$$F_{3/2}(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\gamma} \frac{t^{1/2} dt}{e^{t+\sqrt{\xi^2 + 2M\xi t/\mu}} - 1}, \tag{4}$$

$$T_c(0) = 3{,}31 \frac{n^{2/3}}{M}, \quad \gamma = \frac{\kappa}{T_c} \tag{5}$$

In the case under consideration M/μ=13,92, $\xi = \mu/T_c = 1{,}566 \cdot 10^{12}$ K/T_c,

$\kappa = 10^{-3}K$ ($\approx 10^{-7}$)eV is the value of the wall potential (optical potential) holding neutrons in the trap [8]. The quantity κ also has the meaning of the limiting temperature to which it was possible to cool a neutron in the trap.

Equation of the form (3) is similar to the equation for determining the temperature of a superconducting transition in the bipolaron theory of high-temperature superconductivity [10-11][1] except that in (4) there is a cutoff of the upper limit of integration by the value of the holding potential of the trap.

The expression for $T_c(0)$ defined by (5) is a universal formula for the critical temperature of an arbitrary Bose gas of non-interacting particles. Expression (3) determined the relationship between the critical temperature T_c of a Bose gas of particles interacting with a meson field and the critical temperature $T_c(0)$ of a Bose gas of non-interacting particles.

The Bose condensate occurring in the trap as a whole has a zero momentum P=0. Formally, if the size of the trap were infinite, then the momentum P, being a macroscopic quantity, would retain its value up to the temperature T_c of the order of magnitude of the gap Δ in the spectrum of Bose condensate particles. In reality, the trap has a finite size with a volume of approximately a liter and contains a finite number of particles (of the order of 10^8 neutrons). Therefore, the breakdown of such a Bose condensate will occur already at a gas temperature exceeding the potential of the trap. This circumstance is taken into account by introducing a cutoff γ in the upper limit of the integral in (4).

Thus, the question of the value of the critical temperature of the Bose condensate of dineutrons T_c depending on their concentration n is associated with the solution of problem (3), (4), which will be considered in the next section.

[1] formula (4.9.9) at page 113 in book [10] contains a misprint: instead of the exponent 2/3 it contains 3/2; accordingly, a similar misprint is contained in formula (9.9) at page 147 of book [11]

3. Conditions for the formation of Bose condensate

Currently, the concentrations of ultracold neutrons n of the order of 10^5 cm^{-3} are apparently achievable in terrestrial conditions (according to the project, such values are planned to be obtained at the Kazakh reactor of the Institute of Nuclear Physics by 2030, which will be the highest in the world). However, even for such extreme values of n, the value of $T_c(0)$ from (5), is extremely small. Indeed, using the value: m_n=939,565 MeV for the mass of a neutron and n=10^5 cm^{-3} for the value $T_c(0)$ from (5), we obtain: $T_c(0) = 1{,}72 \cdot 10^{-13}K$. Such a small value of $T_c(0)$ excludes the possibility of experimentally studying the phenomenon of Bose condensation in an ultracold neutron gas.

The situation changes radically if neutrons interact with particles that have a mass gap in their spectrum. From a formal point of view, if the condition P=0 is preserved, then in the dineutron spectrum there is a huge mass gap, determined by (2). According to (3), for the above-mentioned values of the parameters, this gives a value of the order of 10^{11}K for T_c. Hence, in this case, all the particles should fall into the Bose condensate.

This, of course, does not happen, since already at a temperature T higher than $T_\kappa = \kappa$, the particles are no longer held by the trap and the condition P=0 is violated. It follows that the formation of a Bose condensate is little related to the number of dineutrons in the trap and depends entirely on the temperature, which should not exceed κ.

Note that dineutrons, being at T less than T_κ in the condensate state in the lowest energy state, are unable to be captured by the walls of the trap. However, this does not mean that such a condensate will be stable, since supracondensate neutrons will be scattered by the walls of the trap and escape from it. As a result, the number of supracondensate particles will decrease, and their decrease will be compensated by the transition of condensed particles to the supracondensate state until their concentration in the condensate becomes zero.

Our initial assumption about the existence of a bound state of two neutrons in the form of a dineutron is not mandatory. In the case when the binding energy of a dineutron is zero, that is, when there is an ideal gas consisting only of free neutrons, its correlation length will be equal to the Compton wavelength of the nucleon meson coat $\lambda = 1/\mu = 10^{-12}$ cm. This means that any paired spatial configurations can be formed from neutrons in a trap without bringing such pairs closer by distances less than λ. Such pairs can be considered as Bose particles that can undergo Bose condensation and thereby lower the energy of the system at T les than T_κ. In this case, pairs of neutrons whose center of gravity in their Bose condensate is at rest (since all paired particles in the Bose condensate have zero momentum, as does the momentum of the entire Bose condensate as a whole) are chosen as paired Bose particles of an ideal gas of unbound neutrons. The paired states in this case are quite similar to Cooper pairs in conventional superconductors, in which electrons with opposite spins and momenta are correlated. By the same analogy, the instability of a rarefied Fermi gas of nucleons with respect to the formation of a nucleon Bose condensate follows from the fact that at T=0 the energy of the Fermi gas is always positive, and the energy of the Bose condensate is zero. This conclusion is consistent with the general theorem proved by Kohn and Luttinger in 1965: "a many-particle system of fermions weakly interacting by short-range forces will become superfluid even if the interaction is repulsive" [12]. Thus, even with a repulsive potential between neutrons, a neutron gas under the condition that T is less than T_κ will form a Bose condensate with a critical condensation temperature equal to $T_c(0)$.

Let us now consider the opposite case of a very high density of neutron gas, which corresponds to neutron stars and superheavy nuclei (neutron density n is of the order of 10^{38} neutrons per cubic centimeter). In this case, we will assume in (3) $\gamma = \infty$. According to [13], in this case, pairing of nucleons in the singlet state is possible in the inner crust of neutron stars.

According to [14], the temperature of the superfluid component of such stars is of the order of 10^9 K. The calculation performed on the basis of (3) in this case leads to a value of n the order of 10^5 dineutrons per cubic centimeter (see Appendix B). Thus, to realize superfluidity in a neutron star, a very small concentration of dineutrons is required. This, in turn, means that due to such a small n, superfluidity in heavy nuclei is impossible.

4. Discusions

In the introduction, a hypothesis was put forward that explains the leakage of neutrons from the trap due to the formation of dineutrons, which, due to their large characteristic size, have a large scattering cross-section on the walls of the trap. The results obtained allow us to put forward a hypothesis that the formed dineutrons, under the action of gravity, settle to the bottom of the trap, where they accumulate and form Bose-condensate islands with an increased concentration of dineutrons. An increase in the concentration does not lead to an increase in the temperature of the Bose-condensate, since the latter is determined only by the holding potential of the trap. This concentration can be artificially increased by creating quantum nanodots at the bottom of the trap, capable of capturing and holding dineutron clusters in their volume. A model calculation of the characteristics of such nanodots in the case of their capture of non-condensed neutrons yields for the captured neutrons a characteristic energy of the order of a micro-electron-Volt, a characteristic size of the order of tens of nanometers, and their lifetime of the order of a millisecond [15], which coincides with the characteristic thermalization time of ultracold neutrons at a temperature of the order of several millikelvins.

In an ideal Bose gas, its pressure does not depend on the density of the gas. For this reason, the Bose gas can be compressed to a minimum size determined by the size of the dineutron. In the case under consideration, such a compressive force for the neutron Bose condensate is the Earth's gravitational field, which, in the case of an ideal trap surface, leads to the formation of a two-dimensional condensate at the bottom of the trap, with a thickness of the order of an angstrom, and in the presence of defects and a fluctuating potential, to the formation of individual condensate islands at its bottom. Such islands will no longer represent an ideal gas, but a new stable state of matter, which, in the case of an attractive interaction between dineutrons, collapses into a state similar to the state of neutron stars (called neutride or neutronium in popular and science fiction literature).

If the binding energy of a dineutron is positive (i.e. its formation is energetically advantageous) and exceeds T_κ, then the formation of a gas of dineutrons is possible even at a temperature higher than T_κ (an analog of the pseudo-gap phase in high-temperature superconductors [10-11]), which, when the temperature decreases below T_κ, passes into a Bose-condensate state. If the binding energy is below the critical temperature of the Bose-condensate, then the formation of the condensate will occur at a temperature equal to the energy T_κ (similarly to the formation of Cooper pairs and superconductivity at $T = T_c$ in conventional superconductors). As noted above, in any case, a system of weakly interacting fermions, regardless of the type of interaction, cannot remain in the state of a normal liquid up to absolute zero temperatures. The formation of a dineutron gas in the case of a positive binding energy will always occur with the release of energy equal to the energy of synthesis of dineutrons from neutrons, leading to its heating.

The key to observing these effects is to obtain a low temperature T less than T_κ and to increase the neutron concentration in the trap.

Large concentrations are also required to solve fundamental problems: measuring the magnitude of the neutron's electric dipole moment and the neutron lifetime. For this reason, it is hoped that the work on increasing the concentration of ultracold neutrons and lowering their storage temperature will continue. However, new technical ideas are needed for this. As such an idea, we can consider using “artificial” gravity by placing a trap in a centrifuge and thereby creating a centrifugal force that presses the neutrons to the trap wall, or using a dynamic oscillatory mode of the trap's motion. In [16], it was pointed out that it is fundamentally possible to create ultracold neutron concentrations using a laser method, which are several orders of magnitude higher than those currently available and achieve ultralow temperatures much lower than T_κ. In [17], it was proposed to use ultra-cold nanoparticles attached to the surface of the trap to increase the concentration of ultra-cold neutrons and lower the temperature.

As in the case of studying Bose condensates of ultracold atoms in zero gravity, it is of interest to conduct such a study with an ultracold neutron gas. For this purpose, it is necessary to take the neutron source, that is, the reactor, into space. Unlike the experiment under terrestrial conditions, the dineutrons formed in the trap will no longer settle to its bottom. The lifetime of the Bose condensate in this case will be determined only by the lifetime of the supracondensate part of the dineutrons. It can be increased by increasing the size of the trap.

Appendix A. Slow dineutron field theory.

The simplest phenomenological model describing the interaction of two nonrelativistic nucleons with a meson field has the form:

$$H = \frac{1}{2m}\int \nabla_{r_1}\Psi^+\nabla_{r_1}\Psi d^3r_1 d^3r_2 + \frac{1}{2m}\int \nabla_{r_2}\Psi^+\nabla_{r_2}\Psi d^3r_1 d^3r_2 -$$

$$-g\int\big(\varphi(r_1)+\varphi(r_2)\big)\Psi^+\Psi d^3r_1 d^3r_2 + \tfrac{1}{2}\int\{\pi^2 + (\nabla\varphi)^2 + \mu^2\varphi^2\}d^3r\ , \quad (A.1)$$

where $\Psi(r_1, s_1; r_2, s_2)$ is the wave function of two neutrons with coordinates and spins r_1, s_1 and r_2, s_2, m is the mass of the neutron, g –is the constant of the neutron-meson coupling. In the quantum field description, the meson field $\varphi(r)$ is an operator:

$$\varphi(r) = \textstyle\sum_k (2\omega_k^0 V)^{-1/2}\,(b_k + b_{-k}^+)e^{ikr}, \qquad (A.2)$$

$$\pi(r) = \textstyle\sum_k \left(\frac{\omega_k^0}{2V}\right)^{1/2}\,(b_k - b_{-k}^+)e^{ikr},$$

$$\omega_k^0 = \sqrt{\mu^2 + k^2}.$$

where b_k и b_k^+ are the operators of annihilation and birth of mesons.

With the use of (A.1), (A.2) the density of the Hamiltonian H will be:

$$\mathcal{H} = -\frac{\hbar^2}{2m}\Delta_{r_1} - \frac{\hbar^2}{2m}\Delta_{r_2} + \textstyle\sum_k \hbar\omega_k^0 b_k^+ b_k + \qquad (A.3)$$

$$+\textstyle\sum_k [V_k exp(ikr_1)b_k + V_k exp(ikr_2)b_k + H.c.],$$

$$V_k = {g}\Big/{\sqrt{2\omega_k^0 V}},$$

Hamiltonian (A.3) is a non-relativistic quantum field problem of two bodies and coincides in structure with the Hamiltonian of a bipolaron in a polar crystal [10,11]. The simplest model considered by us should be supplemented by taking into account spin interactions by adding a term to H:

$$\int U(r_1, s_1; r_2, s_2)\Psi^{+}\Psi d^3r_1 d^3r_2. \tag{A.4}$$

where s_1, s_2 are the spin and isospin operators. In the case of a contact interaction of Heisenberg-type spins, according to (A.4), a delta-shaped repulsive potential U will be added to the Hamiltonian (A.3), the value of which will determine the stability of the paired state of the nucleons.

It is significant that the translation-invariant theory of bipolarons is based on the same initial Fröhlich Hamiltonian (A.3) as that used in the Bardeen, Cooper, Schrieffer (BCS) theory. The difference between the case of nucleons in a meson field we are considering is only that the BCS theory considers electrons interacting with phonons, while the nucleon-meson field theory considers neutrons interacting with mesons (an application of the BCS theory to neutron matter is given in [18]). The difference in the theoretical approach is that the BCS theory is based on the exclusion of phonon variables from the Hamiltonian and the study of the resulting Hamiltonian, which contains only electron variables. In contrast to the BCS approach, in the TI-bipolaron gas theory, electron variables are excluded from the Hamiltonian. As a result, a Hamiltonian is obtained that depends only on phonon variables. The spectrum of eigenvalues of such a Hamiltonian determines the spectrum of excitations of bipolaron states. The spectrum obtained in this way is then used to describe the statistical properties of an ideal bipolaron gas. In the case under consideration, the role of the TI-bipolaron is played by the dineutron. In this case, ω_k^0 plays the role of the energy gap of the dineutron Bose condensate, the minimum value of which is equal to the meson mass. It is useful to note that the Hamiltonian (A.1) is the simplest model of the interaction of nonrelativistic particles with any type of field $\varphi(r)$. The critical temperature T_c described by (3)-

(5), does not depend at all on the value of the interaction constant g and is determined by the value of the gap corresponding to such a field for which the gap is minimal. A candidate for the role of such a field could be the neutrino field (the dineutron model based on the interaction of neutrons with a neutrino field was considered in [19]), for which the upper estimate of the mass is 0.3 eV. The photon field has an even smaller gap, or rather its absence (the interaction of a neutron with a photon, even in the absence of a neutron spin, will be different from zero due to the presence of an electric dipole moment). In the latter case, the temperature of the Bose-condensate of dineutrons will be equal to $T_c(0) = 1{,}72 \cdot 10^{-13} K$. Due to the weak interaction of neutrons with neutrino and photon fields, the establishment of thermal equilibrium in a neutron gas by such interactions requires so much time that their detailed consideration is hardly of interest from the point of view of terrestrial experiments.

An important point is that a dineutron in a Bose condensate cannot scatter on the walls of the trap, since this is equivalent to the appearance of excitation in it – the escape of one of the particles from the condensate and an increase in the energy of the condensate.

Appendix B. Solution of equation (3).

1.*Ultracold neutrons*

Since the value of μ is very large ($\mu = 1{,}566 \cdot 10^{12} K$), we set $\mu \gg T_c$. Due to the fact that the value of κ is small, we will also assume $\gamma = \kappa / T_c \ll 1$.

In this case, for $F_{3/2}(\xi)$ from (4) we obtain:

$$F_{3/2}(\xi) \approx \frac{2}{\sqrt{\pi}} e^{-\xi} \int_0^{\gamma} t^{1/2} dt = \frac{4}{3\sqrt{\pi}} \gamma^{3/2} e^{-\xi} \qquad \text{(B.1)}$$

Expression (B.1), according to the assumptions made, is valid for $\xi \gg 1$. In the case of $\xi = 0$, for $F_{3/2}(0)$ we have:

$$F_{3/2}(0) = \frac{2}{\sqrt{\pi}} \int_0^{\gamma} \frac{t^{1/2} dt}{e^t - 1} = \frac{2}{\sqrt{\pi}} \int_0^{\gamma} \frac{dt}{t^{1/2}} = \frac{4}{\sqrt{\pi}} \gamma^{1/2} \qquad \text{(B.2)}$$

Then from (3) we get:

$$T_c \approx \left(\frac{3}{\gamma}\right)^{2/3} e^{2\xi/3} T_c(0) \tag{B.3}$$

Assuming in (B.3)

$$T_c = 2\,{}^{\mu}\!/_{W} \tag{B.4}$$

We obtain from (B.3) Lambert equation for the function $W(z)$:

$$z = We^{W} \tag{B.5}$$

where:

$$z = \frac{2\mu\kappa^2}{9T_c^3(0)}$$

for the parameters we use, the value $z{\sim}10^{44}$. For this z from (B.5) we get: $W{\sim}10^2$. As a result, for T_c from (B.4) we have: $T_c{\sim}10^{11}K$. Thus, the assumption we made, that $\xi = \mu/T_c \gg 1$ is justified.

2. Neutron star.

Even if we take the upper estimate for the Bose condensation temperature of dineutrons in a neutron star $T_c = 2 \cdot 10^{10}K$, then for the concentration of dineutrons in a neutron star n using the numerical solution of (3) with $\gamma \gg 1$ we obtain the value $n \approx 10^5$ dineutrons/cm^3. This estimate changes little for the assumed range of possible values of $T_c{\sim}10^8 \div 10^{10}K$.

References

[1] N. Dzysiuk and I.M. Kadenko, Candidate-nuclei for observation of a bound dineutron. Part II: The (n, $2n$ +n) and (γ, $2n$) nuclear reactions // Nuclear Physics A, 2025, 1053, 122961; doi https://doi.org/10.1016/j.nuclphysa.2024.122961

[2] S.Huang, Z.Yang, Neutron clusters in nuclear systems // Front. Phys., 2023, 11, 1233175; doi: 10.3389/fphy.2023.1233175

[3]. I. Kadenko, Possible observation of the dineutron in the $^{159}Tb(n,^{2}n)^{158g}Tb$ nuclear reaction // EPL, 2016, 114, 42001; doi https://doi.org/10.1209/0295-5075/114/42001

[4] A. Corsi, Y. Kubota, J. Casal, M. Gómez-Ramos, A.M. Moro, G. Authelet, H. Baba, C. Caesar, D. Calvet, A. Delbart, M. Dozono, J. Feng, F. Flavigny, J.-M. Gheller, J. Gibelin, A. Giganon, A. Gillibert, K. Hasegawa, T. Isobe, Y. Kanaya…J. Zenihiro, Searching for universality of dineutron correlation at the surface of Borromean nuclei // Physics Letters B, 2023, 840, 137875; doi https://doi.org/10.1016/j.physletb.2023.137875

[5] A. Spyrou, Z. Kohley, T. Baumann, D. Bazin, B. A. Brown, G. Christian, P. A. DeYoung, J. E. Finck, N. Frank, E. Lunderberg, S. Mosby, W. A. Peters, A. Schiller, J. K. Smith, J. Snyder, M. J. Strongman, M. Thoennessen, and A. Volya, First Observation of Ground State Dineutron Decay: ^{16}Be // Phys. Rev. Lett., 2012, 108, 102501; doi http://doi.org/10.1103/PhysRevLett.108.102501

[6] Y. Kubota, A. Corsi, A., G. Authelet, et. al., Surface localization of the dineutron in ^{11}Li // Phys. Rev. Lett., 2020, 125, 252501; doi https://doi.org/10.1103/PhysRevLett.125.252501

[7] A.V. Strelkov, History of the discovery of ultracold neutrons // LNP-JINR, 1996; doi http://nuclphys.sinp.msu.ru/ucn/hist.htm

[8] A.P. Serebrov, Neutron lifetime measurements using gravitationally trapped ultracold neutrons // Phys.-Usp., 2005, 48, 867; doi 10.1070/PU2005v048n09ABEH003536

[9] A.P. Serebrov, Disagreement between measurements of the neutron lifetime by the ultracold neutron storage method and the beam technique // Phys.-Usp., 2019, 62, 596–601; doi 10.3367/UFNe.2018.11.038475

[10] V.D. Lakhno, High Temperature Superconductivity. Bipolaron Mechanism, De Gruyter, Berlin, 2022; doi https://doi.org/10.1515/9783110786668

[11] V.D. Lakhno, Mathematical foundations of the translation-invariant bipolaron theory of superconductivity, M.: KIAM them. M.V. Keldysha, 2021, 292 pages doi https://doi.org/10.20948/mono-2021-lakhno, https://keldysh.ru/e-biblio/lakhno/

[12] W. Kohn, J. Luttinger, New Mechanism for Superconductivity // Phys. Rev. Lett., 1965, 15, 524; doi https://doi.org/10.1103/PhysRevLett.15.524

[13] S. L. Shapiro, S.A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars (New York: Wiley, 1983)

[14] P.S. Shtern, D.G. Yakovlev, Superfluid neutron stars // Phys.-Usp., 2012, 55, 935; doi 10.3367/UFNe.0182.201209g.1006;

[15] H.Tang, G. Wang, P. Cappellaro , μeV - Deep Neutron Bound States in Nanocrystals // ACSNano, 2024, 18, 12, 906363 – 9070; doi: 10.1021/acsnano.3c12929

[16] L.A. Rivlin On the laser method of production of ultracold neutrons // Quantum Electron, 2011, 41, 659; doi 10.1070/QE2011v041n07ABEH014514

[17] V.V. Nesvizhevskii, Quantum states of neutrons in a gravitational field and the interaction of neutrons with nanoparticles // Phys.-Usp., 2003, 46, 93; doi 10.1070/PU2003v046n01ABEH001347

[18] M. Matsuo, Spatial structure of neutron cooper pair in low density uniform matter // Phys. Rev. C, 2006, 73, 044309; doi:10.1103/PhysRevC.73.044309

[19] Y.L. Rutis, Neitrinniy kataliz reaktsii sliyaniya yader v kholodnm vodorode, Prikladnaya Fizika, 2010,1, 21-30

Contents