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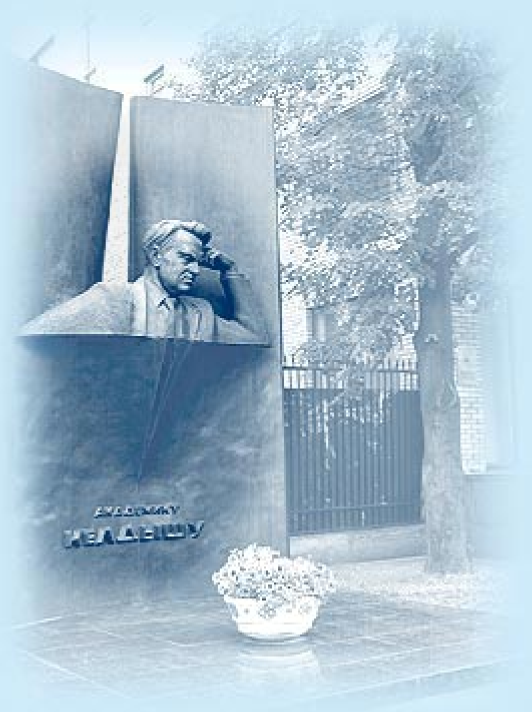

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V.D. Lakhno

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Keldysh Institute of Applied Mathematics

of Russian Academy of Sciences

Lakhno V.D.

On the possibility of creating neutron matter in terrestrial conditions

Moscow – 2025

В.Д. Лахно

О возможности создания нейтронной материи в земных условиях

Обсуждается вопрос о возможности образования нейтронной материи, представляющей собой бозе-конденсат динейтронов. Считается, что существование стабильных динейтронов подтвеждается наличием нейтронной аномалии в ультрахолодном нейтронном (УХН) газе. Показано, что время образования динейтрона в УХН газе в этом случае составляет примерно 27 часов. Предложена схема создания накопительной ловушки для динейтронов. Сделан вывод о возможности образования бозе-конденсата УХН газа при сколь угодно малой концентрации нейтронов в ловушке при температуре Т ниже критической нейтронного бозе-конденсата: $T_c = 10^{-3}$. Показано, что максимальная плотность нейтронной материи в бозе-конденсате на четыре порядка превышает плотность обычного вещества.

Ключевые слова: динейтрон, бозе-конденсат, ультрахолодные нейтроны, нейтронная аномалия, плотность нейтронной материи

V. D. Lakhno

On the possibility of creating neutron matter in terrestrial conditions

The question of the possibility of formation of neutron matter, which is a Bose condensate of dineutrons, is discussed. It is believed that the existence of stable dineutrons is confirmed by the presence of a neutron anomaly in ultracold neutron (UCN) gas. It is shown that the time of dineutron formation in UCN gas in this case is approximately 27 hours. A scheme for creating a storage trap for dineutrons has been proposed. A conclusion is made about the possibility of formation of a Bose condensate of UCN gas at an arbitrarily small concentration of neutrons in the trap at a temperature T below the critical temperature Tc of the neutron Bose condensate: $T_c = 10^{-3}$. It is shown that the maximum density of neutron matter in the Bose condensate is four orders of magnitude higher than the density of ordinary matter.

Key words: dineutron, Bose condensate, ultracold neutrons, neutron anomaly, neutron matter density

1. Introduction

The question of creating stable neutron matter runs up against the dineutron problem. This is due to the fact that the creation of a neutron Bose condensate requires the presence of paired neutron states, i.e. dineutrons, in the ultracold neutron (UCN) gas. The existence of such states has currently been experimentally proven only in nuclear decay reactions as virtual short-lived states [1]-[6]. In the low-energy limit, the question of the existence of stable dineutrons remains open. In the work of the author [7], it was suggested that dineutrons may form in the UCN gas. Their formation can explain the presence of a neutron anomaly in the UCN gas – an unexplained leakage of neutrons from a neutron trap. This raises a number of fundamental questions: 1) If dineutrons are formed in the UCN gas, then their stability is required to observe the formation of their Bose condensate 2) The number of dineutrons in the UCN gas is only a small part of the neutrons in the trap. Is the formation of a condensate possible in this case? 3) If a dineutron condensate is formed, how can it be detected? 4) Irrespective of the question of the Bose condensate, the question arises how it is possible to detect a dineutron at all even in the case of its stability. Although, by analogy with the case of cold atoms, when their Bose condensate is possible even in the case of half-integer spins (Fermi gas), as is the case with neutrons, the fundamental answer to the question of the possibility of the formation of a Bose condensate is positive (according to the mechanism of formation of Cooper pairs in a superconductor), the practical implementation of such a Bose condensate is far beyond the experimental capabilities (section 5). In section 2 we will answer question 1), in section 3 question 2), in section 4, question 3 will be answered. Question 4) is discussed throughout the text of the paper. Section 6 discusses some of the implications of the possible existence of neutron matter.

2. Stability and time of formation of the dineutron

In 1930, saving "exchange statistics and the law of conservation of energy," W. Pauli predicted the existence of neutrinos in the process of beta decay of the neutron:

$$n \rightarrow p^{+} + e^{-} + \bar{\nu}_e$$

This prediction was experimentally confirmed only in 1956. Hence, it was concluded that the dineutron is unstable even if its existence were energetically beneficial. For this reason, it is not surprising that the discovery of dineutrons is even more difficult than the discovery of neutrinos, since it is not clear how such a particle can be created.

According to the field meson theory of nuclear forces, the formation of a dineutron can be described as a result of the interaction of nucleons with a meson field. There is a complete analogy with the bipolaron theory of the interaction of two electrons with a scalar field (phonon field) in condensed matter physics. If such a formation is energetically advantageous, it should be stable with respect to its decay into individual neutrons. Accordingly, the decay of one of the neutrons in a dineutron would lead to its destruction, which is prohibited by the law of conservation of energy in the reaction:

$$^{2}n + E_{dn} = n + n \tag{1}$$

where E_{dn} is the binding energy of the dineutron. Thus, the nature of the stability of the dineutron is the same as that of the neutrons in the nucleus. The difference here is purely quantitative – the binding energy of the dineutron is much lower than that in the nucleus. Note that, according to [7], even in the case of repulsive interaction of neutrons, when their bound state is absent, the formation of a condensate makes their existence stable both with respect to decay into individual neutrons and with respect to their beta decay. The situation here is close to the case of the formation of a Bose condensate in the Fermi gas of ultracold atoms, realized experimentally. We will consider this issue in Section 5.

If dineutrons are indeed formed in UCN, then the question of the time of their formation is of great interest. If we assume that there are only two channels for the extinction of neutrons in a neutron trap: a) due to beta decay, b) due to the formation of dineutrons, then the equation which determines the dynamics of the number of neutrons in the trap will have the form:

$$\frac{dN}{dt} = -\left(k_\beta + k_d\right)N$$

$$k = k_\beta + k_d = 1/\tau = {}^{1}/_{\tau_\beta} + {}^{1}/_{\tau_d}, \qquad (2)$$

where $\tau = 879$ sec is the lifetime in the UCN trap, $\tau_\beta = 887$ sec is the lifetime of a neutron in beta decay in a beam experiment [8]. Hence, the formation time of a dineutron τ_d is:

$$\tau_d = \frac{\tau\tau_\beta}{\tau_\beta - \tau} = \frac{879 \cdot 887}{887 - 879}\text{сек} = 27 \text{ hours} \qquad (3)$$

The results obtained allow us to estimate the number of dineutrons in the trap from the equation that determines the kinetics of dineutron formation:

$$dN_d/dt = k_d N, \qquad (4)$$

where N_d in the number of dineutrons.

From (2) and (4) it follows that:

$$dN_d/dN = -k_d/\left(k_\beta + k_d\right) \approx -k_d/k_\beta \ ,$$

It follows that:

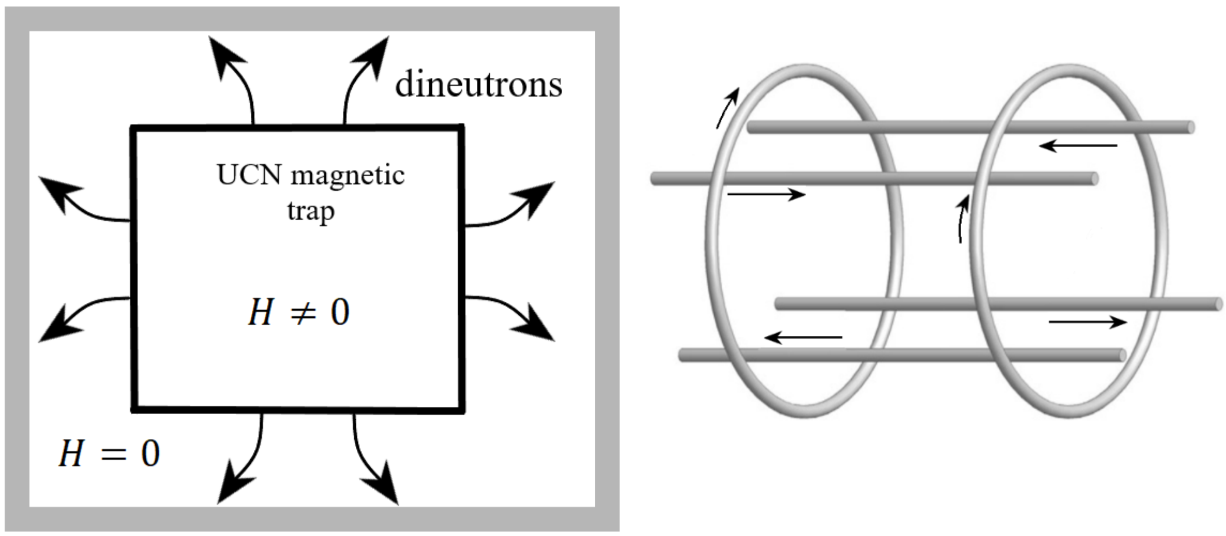
$$N_d(t) = k_d\left(N(0) - N(t)\right)/k_\beta \qquad (5)$$

From (5) it follows that $N_d(\infty) = k_d\, N_0/k_\beta$, $k_d/k_\beta = 9.1 \times 10^{-3}$. If the concentration of neutrons in the trap is maintained constant $N = N_0 = const$, then: $N_d = k_d N t$, k_d = 1/(27 hours) – the concentration of dineutrons increases linearly with time and in 27 hours their number will become equal to N_0.

Note that if, in addition to the formation of dineutrons, there are other channels for neutron leakage from the trap, then the estimate of the time for the formation of dineutrons will increase.

The obtained estimates allow us to propose a technical method for obtaining high concentrations of dineutrons. Fig. 1 schematically shows the method of accumulation in a material trap of ultracold dineutrons (UCDN). Neutrons initially enter a magnetic trap, which holds the neutrons due to their spin. Dineutrons formed in the magnetic trap have a spin equal to zero and freely exit the magnetic trap, accumulating in the material trap. Their presence can be detected by scintillations using conventional neutron sensors. The number of dineutrons in the UCDN material trap after all neutrons in the magnetic trap have undergone beta decay, according to (5), will be of the order of one percent of the initial number of neutrons in the magnetic trap.

A material trap UCDN **UCN magnetic trap**



a) b)

Fig. 1. a) A UCN magnetic trap located inside a material trap holds neutrons. The material trap holds dineutrons. b) The simplest version of a magnetic trap: longitudinal currents form a quadrupole magnetic field, holding neutrons transverse to the axis, while transverse ring currents hold neutrons along the trap axis (Ioffe-Pritchard trap)

It is difficult to answer the question about the further fate of the formed dineutrons (see Section 6). The optical potential holding the neutrons in the material trap can be significantly lower for dineutrons, since, being a spinless particle, the dineutron will not interact with the repulsive spin part of the nuclear potential of the trap walls. Therefore, even lower temperatures may be required to trap the dineutrons than to trap the neutrons. In this case, however, the formation of a Bose condensate of dineutrons becomes possible. The formation of a Bose condensate in a material trap can explain the reason why the measurement of the neutron lifetime in a magnetic trap coincides with the measurement of this time in a material trap. In a magnetic trap, the anomalous loss of neutrons is associated with the formation of dineutrons and their automatic disappearance from the trap. The formation of a Bose condensate in this case is impossible. In a material trap, the formation of dineutrons leads to the formation of their Bose condensate, which cannot be recorded by conventional sensors, since the Bose condensate particles, as shown in [7], cannot leave it. This is interpreted as their anomalous loss, although the dineutrons remain in the trap.

3. The possibility of the formation of a dineutron Bose condensate

Being bosons, the dineutrons formed in the trap can form a Bose condensate. In [7], the following expression for the critical temperature of the Bose condensate formation was used to estimate the temperature of the dineutron condensate:

$$T_c = \left(F_{3/2}(0)/F_{3/2}(\xi)\right)^{2/3} T_c(0) \tag{6}$$

$$F_{3/2}(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\gamma} \frac{t^{1/2}dt}{e^{t+\sqrt{\xi^2+2M\xi t/\mu}}-1}, \tag{7}$$

$$T_c(0) = 3.31\frac{n^{2/3}}{M}, \quad \gamma = \frac{\kappa}{T_c}, \tag{8}$$

$$N_1/N = (T/T_c)^{3/2}, \quad N_0/N = 1-(T/T_c)^{3/2},$$

In the case under consideration M/μ=13.92, $\xi = \mu/T_c = 1.566 \cdot 10^{12}$ K/T_c, (μ is the meson mass, $M = 2m$, m is the neutron mass, $\kappa = 10^{-3}K$ ($\approx 10^{-7}$eV) is the value of the wall potential (optical potential) holding the neutrons in the trap [7], [8], N_0 and N_1 are the numbers of condensate and supercondensate Bose particles, respectively. The quantity κ also has the meaning of the temperature limit to which the neutrons in the trap could be cooled. (specific values of κ for different substances can be found, for example, in the paper "Ultracold Neutrons", Physical Encyclopedia, 1998 in Russian)

The equation of the form (6) is similar to the equation for determining the temperature of a superconducting transition in the bipolaron theory of high-temperature superconductivity [9-10], except for the existing cutoff of the upper limit of integration in (7) by the value of the holding potential of the trap.

The expression for $T_c(0)$, determined by (8), is a universal formula for the critical temperature of an arbitrary Bose gas of non-interacting particles. Expression (6) determines the relationship between the critical temperature of the Bose gas particles T_c, interacting with the meson field and the critical temperature $T_c(0)$ of the Bose gas of non-interacting particles.

Equations (6), (7) have an interesting feature: the value of the critical temperature T_c determined by them is practically independent of the neutron concentration in the trap (see Appendix). This leads to important conclusions: firstly, the Bose condensate of neutrons in the trap can form at an arbitrarily low concentration, and secondly, at any concentration of dineutrons in the trap, the temperature of their Bose condensation will be determined not by the values given in the Appendix, but by the value of the trap potential, equal to 10^{-3} K.

4. Peculiarities of neutron scattering by dineutron Bose condensate

Neutron scattering can be used to identify the formation of a dineutron Bose condensate in a trap.

Neutron scattering is a powerful method for studying the properties and structure of crystals, widely used in modern physics. Using neutrons, one can determine the atomic structure, reveal the magnetic structure, and obtain information about the spectral characteristics of various excitations in crystals, liquids, etc.

The neutron scattering method plays one of the key roles in the study of high-temperature superconductors (HTSC). In [9]-[10] the author outlined the translation-invariant bipolaron theory of superconductivity, which is based on the phenomenon of Bose condensation of TI bipolarons. The role of Cooper pairs in this model is played by TI bipolarons, which are bound states of spatially delocalized electrons with a small correlation length. The theory developed in [9], [10] explains a large number of experiments on the thermodynamic and transport characteristics of high-temperature superconductors, Josephson tunneling, angle-resolved photoelectron spectroscopy, Raman scattering, etc. In this paper, we consider the features that arise during neutron scattering by a dineutron Bose condensate, which can be detected experimentally. The consideration is based on an analogy with neutron scattering by a bipolaron condensate.

In the approach we are considering, based on the Bose condensation of dineutrons, a neutron striking the system under study (in our case, such a system is a dineutron condensate) takes it out of the state of thermodynamic equilibrium, and at the same time the state of the incident neutron itself changes. By recording the energy and momentum of the incident scattered neutron, we obtain information about the excitations that are possible in the system under study.

As is known, the cross section of inelastic scattering σ on a system of particles of one type at a temperature $T = \beta^{-1}$ per unit solid angle Ω has the form:

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{M_n^2}{(2\pi)^3\hbar^4}\frac{k_f}{k_i}\left|V\left(\vec{k}\right)\right|^2 N\frac{S\left(\vec{k},\omega\right)}{1-e^{-\beta\omega}} \quad (9)$$

where N is the number of particles in the system, M_n is the mass of the neutron, $\hbar\vec{k} = \hbar\left(\vec{k}_i - \vec{k}_f\right)$ is the value of the momentum transferred during scattering, where $\vec{k}_i$ и and $\vec{k}_f$ are the initial and final momenta of the neutron, respectively; ω is the energy transferred as a result of scattering; $V\left(\vec{k}\right)$ is the Fourier component of the interaction potential of the neutron with the scattering particle; the quantity $S\left(\vec{k},\omega\right)$ involved in (9) is a dynamic structural factor that is related to the density-density correlation function by the relation:

$$S\left(\vec{k},\omega\right) = \int d^3r \int_{-\infty}^{\infty}\frac{dt}{\hbar}e^{-ikr+i\omega t/\hbar}\langle[\rho(r,t),\rho(0,0)]\rangle \quad (10)$$

where $\rho(r,t) = \Psi^+(r,t)\Psi(r,t)$ is the particle density operator in the Heisenberg representation, $\Psi(r,t) -$ is the dineutron wave function), $\langle\dots\rangle$ is the thermodynamic average.

In the case of a conventional ideal Bose gas, whose particles have the spectrum $\varepsilon_k = k^2/2M$, the structure factor $S\left(\vec{k},\omega\right)$ was calculated in [11]-[13]. In the case of an ideal gas of dineutrons with the spectrum $\varepsilon_0 = 0$ and $\varepsilon_{k\neq 0} = \omega_0 + k^2/2M$

(where ω_0 is the optical potential of the trap, which plays the role of the optical phonon frequency), which corresponds to the distribution function $F(\vec{k})$:

$$F(\vec{k}) = n_0(\vec{k})\delta(\vec{k}) + [exp(\beta\varepsilon_k) - 1]^{-1}, \quad T < T_c$$

$$F(\vec{k}) = [exp\beta(\varepsilon_k - \mu_{chem}) - 1]^{-1}, \quad T \geq T_c$$

where $n_0(\vec{k})$ is the concentration of dineutrons in the Bose condensate, μ_{chem} is the chemical potential, the expression for the structure factor (10) takes the form:

$$S(\vec{k},\omega) = S_0(\vec{k},\omega) + S_1(\vec{k},\omega), \tag{11}$$

$$S_0(\vec{k},\omega) = n_0\left[\left(1 + F(\vec{k})\right)\delta(\omega - \omega_0 - k^2/2M) - F(\vec{k})\delta(\omega + \omega_0 + k^2/2M)\right],$$

$$S_1(\vec{k},\omega) = \frac{2\pi M}{\hbar^4\beta k(1 - e^{-\beta\omega})}$$

$$\cdot \ln\frac{1 - \exp\{-\beta[\omega_0 + \beta M\,(\omega + k^2/2M)^2/2k^2] - \tilde{\mu}\}}{1 - \exp\{-\beta[\omega_0 + \beta M\,(\omega - k^2/2M)^2/2k^2] - \tilde{\mu}\}},$$

$$\tilde{\mu} = \mu_{chem} - E_{dn}; \qquad \tilde{\mu} = 0, \qquad T \leq T_c; \qquad \tilde{\mu} = \tilde{\mu}(T), \qquad T > T_c.$$

It follows that for $T \to 0$, when $F(\vec{k}) \to 0$, there are no processes with energy absorption and only processes with energy transfer remain.

In expression (9), the form factor $S_0(\vec{k},\omega)$ determines the excitations of individual dineutrons in the Bose condensate, and $S_1(\vec{k},\omega)$ is the contribution of dineutrons in the supra-condensate state.

From (11) it follows that when $\omega > 0$ the form factor has two peaks. Fig. 2 shows the function $S(\vec{k},\omega)$ for a certain fixed value of $\vec{k}$,

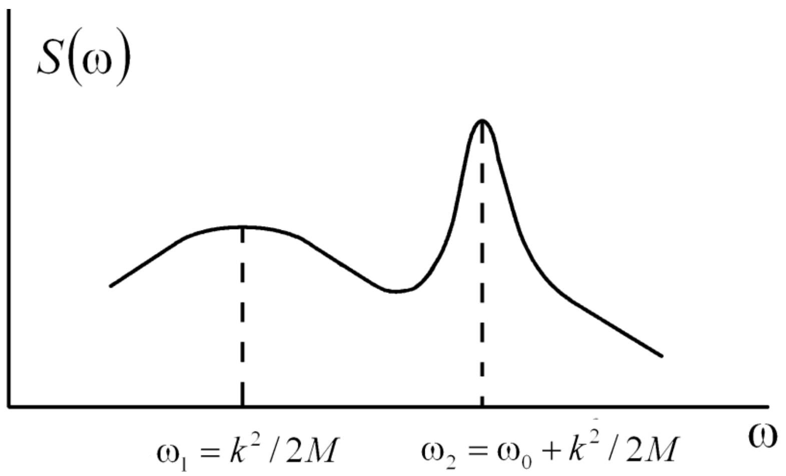


Fig. 2. Dependence of the scattering form factor on the neutron energy.

which has one (blurred) peak at the energy $\omega = \omega_1 = k^2/2M$ (it is determined by $S_1(\vec{k}, \omega)$), and the second (sharper) peak at the energy $\omega = \omega_2 = \omega_0 + k^2/2M$ (it is determined by $S_0(\vec{k}, \omega)$). The energy difference between the two peaks of ω_0, according to the TI-bipolaron theory, determines the size of the gap in the condensate. At $\omega_0 = 0$ the two peaks merge into one, which corresponds to an ideal Bose gas.

It follows from (11) that at $T = T_c$ the maximum at $\omega = \omega_2$ disappears, since in this case $n_0 = 0$. The maximum at $\omega = \omega_1$ is also preserved at $T > T_c$ and disappears only at $T = T^*$, where T^* is the temperature of the transition of the condensate to the pseudophase. At $T = 0$, on the contrary, the maximum at $\omega = \omega_1$, disappears, and the maximum at $T = \omega_2$ becomes maximally sharp and intense.

In a real experiment, the peak in neutron scattering that appears at $T < T_c$ is obtained by subtracting the scattering in the normal and condensed phases. Thus, the energy $\omega_2 = \omega_0 + k^2/2M$ is compared to the value of the condensate gap, since the peak with $\omega = \omega_1$ is automatically subtracted in this approach. This can lead to both an overestimation of the gap value and a distortion of its angular dependence.

It should be noted that the neutron scattering method was actively used to determine the presence of a superfluid component in ${}^4He$, which presumably represents a Bose-condensate state, into which some helium atoms pass at $T < T_c$ [11]-[13], [14], [15], as was first suggested in [16].

The theory of neutron scattering on the Bose-condensate formed at $T < T_c$ which is considered as an ideal gas, leads to the conclusion that there is a sharp peak caused by the presence of the condensate, the position of which coincides with the position of the maximum of the broad peak associated with the supercondensate excitations (which corresponds to $\omega_0 = 0$). This circumstance causes great difficulties in isolating the Bose-condensate contribution. According to the above, this difficulty is absent in the case of a dineutron Bose condensate, in which the corresponding peaks are spaced by ω_0, which makes it promising to use the inelastic scattering method to study the properties of a dineutron Bose condensate.

5. Bose condensate of UCN in the absence of a stable state of a free dineutron

Above we considered the case when the binding energy of the dineutron is positive and the dineutron is a stable particle that exists both in the absence of the Bose condensate and in its presence. If the binding energy of the dineutron is insufficient for the formation of a free stable particle, then the formation of dineutrons will occur simultaneously with the formation of their Bose condensate. The existence of a pseudogap phase in this case becomes impossible. This is exactly the case in the BCS theory of superconductivity of conventional superconductors. The temperature of the transition of UCN gas to the condensate state will be determined by the concentration of UCN in the trap by the relation:

$$T_c = 0.218\, E_F \tag{12}$$

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = \frac{\hbar^2}{2m}(3\pi^2 \cdot 2n_B)^{2/3}$$

$$T_c = 3.31 \cdot \left(n_B^{2/3}\hbar^2\right)/M_B = 3.31 \cdot \left(n_B^{2/3}\hbar^2\right)/2m,$$

where n is the concentration of UCN, n_B is the concentration of Bose particles (i.e. dineutrons), M_B is their mass, E_F is the Fermi energy of UCN gas:

$$T_c < T < E_F - \text{Fermi gas} \tag{13}$$

$$T < T_c - \text{Bose condensate.} \tag{14}$$

In particular, for $n = 10^5 cm^{-3}$, $T_c \approx 1.72 \cdot 10^{-13} K$. It follows that for the experimental realization of a Bose gas from unstable dineutrons it is necessary to increase the concentration of the UCN gas by several orders of magnitude.

It should be noted that the fact of the formation of a Bose condensate from particles that are fermions by nature is a consequence of the general theorem proved by Kohn and Lutinger: “A multiparticle system of fermions weakly interacting by short-range forces will become superfluid, even if the interaction is repulsive” [17].

According to (13), when the temperature in the UCN gas decreases, the Fermi gas will first form, and when it decreases further, according to (14), the Bose gas will form. Here, various complex effects are possible in the behavior of the Bose gas of dineutrons in the presence of a degenerate gas of neutrons, usually observed in mixtures of bosonic and fermion gases of ultracold atoms [18].

6. Neutron matter

The phrase neutron matter, as a substance containing mainly neutrons, is usually associated with objects such as neutron stars. Currently, it is generally accepted that the creation of neutron matter in terrestrial conditions is impossible due to: 1) its extreme density, 2) neutron matter decays rapidly, 3) only the super-strong gravitational field of neutron stars can hold neutron matter.

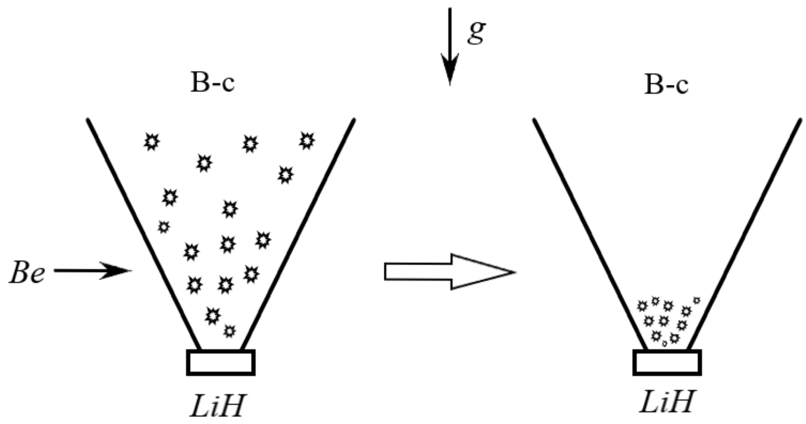


Fig. 3. Bose condensation of dineutrons in a trap

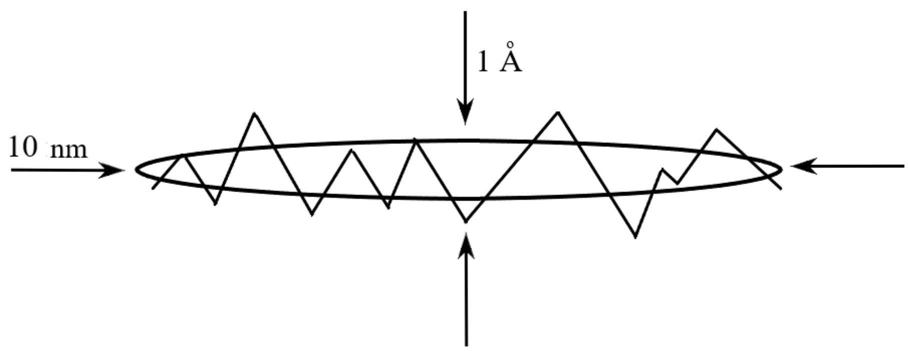


Fig. 4. Formation of a localized dineutron state in a LiH nanocrystal at the bottom of a trap.

This paper shows that this view is incorrect. Fig. 3 shows a scheme of the production of neutron matter as a result of the formation of a Bose condensate from UCN.

Having a zero bulk modulus and infinite compressibility under the action of the gravitational field, the dineutron gas collects at the bottom of the trap. Figure 4 shows the profile of the nanocrystal potential and the localized state into which all the dineutrons in the trap fall. Computer modeling of the characteristics of neutron states in a LiH nanocrystal was carried out in [19]: for the neutron energy, a value of the order of a micro-electron-volt was obtained, the distribution of neutron density by height was of the order of an angstrom and in diameter 10 nm.

Let us estimate the maximum density of the neutron matter obtained in this way. The characteristic spatial size of the optical potential a is determined from the relation: $a = \hbar/\sqrt{2m\kappa}$ and for $\kappa = 10^{-7}$ eV it is $a = 100$ A. The maximum value of the mass of the neutron island m_1, held by the potential κ $m_1 ga = \kappa$, is found from the equality $m_1 1ga = \kappa$, where g is the acceleration of gravity. It follows that the maximum value of the mass of a neutron island is 1.6×10^{-16}g. With a neutron mass of 1.67×10^{-24}g the number of neutrons in the island will be 10^8 neutrons. The volume of the island, according to Fig. 4, is $V_1 = 1 \times 100A \times 100A = 10^{-20}$ cm^3. Hence, the maximum density of neutron matter will be: $\rho_0 = 1.6 \times 10^4$ g/cm^3 or 16 kilograms per cubic centimeter. The neutron matter thus obtained will, however, be unstable relative to an even denser state in which it is all collected in a single dineutron, the size of which is determined by its binding energy E_{dn}. With such a high density of the formed state, its further retention in the trap becomes impossible and, under the influence of the gravitational field, it will move toward the center of the earth, possibly capturing the nuclei of the surrounding matter. The description of this situation, however, goes far beyond the scope of this paper.

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Appendix Numerical solutions of (6), (7) equations

$T_c(\xi), K$	$T_c(0), K$	ξ
1.57×10^{12}	5.9×10^{10}	1.
3.13×10^{12}	2.35×10^{11}	0.5
4.7×10^{12}	4.9×10^{11}	0.33
6.26×10^{12}	8.06×10^{11}	0.25
7.83×10^{12}	1.17×10^{12}	0.2
9.4×10^{12}	1.57×10^{12}	0.17
1.1×10^{13}	2.01×10^{12}	0.14
1.25×10^{13}	2.48×10^{12}	0.13
1.4×10^{13}	2.98×10^{12}	0.11
1.57×10^{11}	7.8×10^{6}	10.
7.83×10^{10}	4723	20.
5.22×10^{10}	3.93	30.
3.92×10^{10}	0.0037	40.
3.13×10^{10}	3.78×10^{-6}	50.
2.61×10^{10}	3.99×10^{-9}	60.
2.24×10^{10}	4.34×10^{-12}	70.
1.96×10^{10}	4.82×10^{-15}	80.
1.74×10^{10}	5.45×10^{-18}	90.
1.57×10^{10}	6.23×10^{-21}	100.
7.83×10^{9}	3.45×10^{-50}	200.
5.22×10^{9}	2.56×10^{-79}	300.
3.92×10^{9}	2.14×10^{-108}	400.
3.132×10^{9}	1.91×10^{-137}	500.
2.61×10^{9}	1.77×10^{-166}	600.
2.24×10^{9}	1.69×10^{-195}	700.
1.96×10^{9}	1.65×10^{-224}	800.
1.74×10^{9}	1.63×10^{-253}	900.
2.2×10^{10}	1.75×10^{-12}	71.3
2.17×10^{10}	9.98×10^{-13}	72.2
2.15×10^{10}	5.63×10^{-13}	73
2.12×10^{10}	3.13×10^{-13}	73.9
2.1×10^{10}	1.72×10^{-13}	74.7
2.07×10^{10}	9.31×10^{-14}	75.6
2.05×10^{10}	4.97×10^{-14}	76.6
2.02×10^{10}	2.6×10^{-14}	77.5
1.99×10^{10}	1.35×10^{-14}	78.4

The solution used in [7] is highlighted.

*results obtained by A.N. Korshunova

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