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Femtosatellites Swarm Motion Estimation Using Measurements of Communication Signal Magnitude

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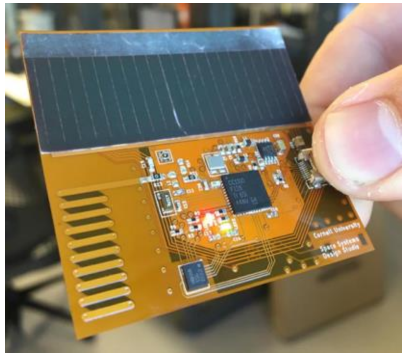
Introduction

- Femtosatellite advantages
  - Multiple spatial measurements in orbit
  - Numerous of laboratory prototypes
  - Cheap production and launch
  - Lot of femtosats can be launched together
- Motivation for novel algorithm for relative navigation
  - Low power supply
  - GNSS-based navigation is inappropriate
  - Angles only camera-based navigation is characterized by unobservability

<u>Communication antenna-based navigation should be investigated</u>



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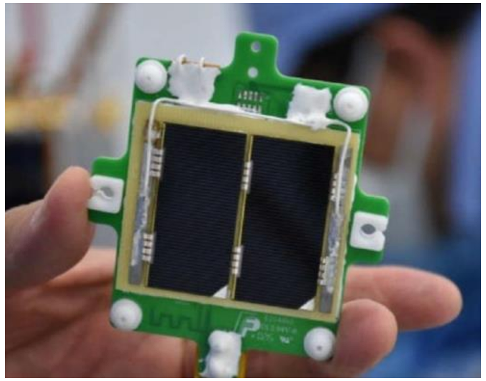

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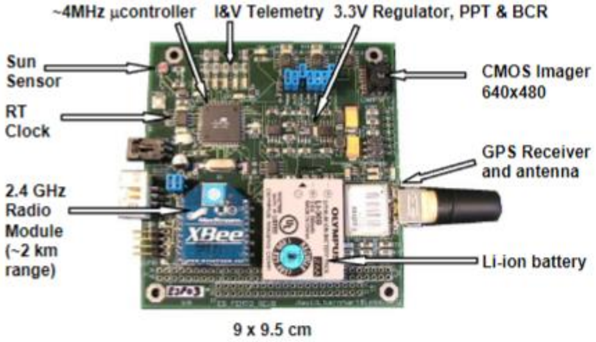

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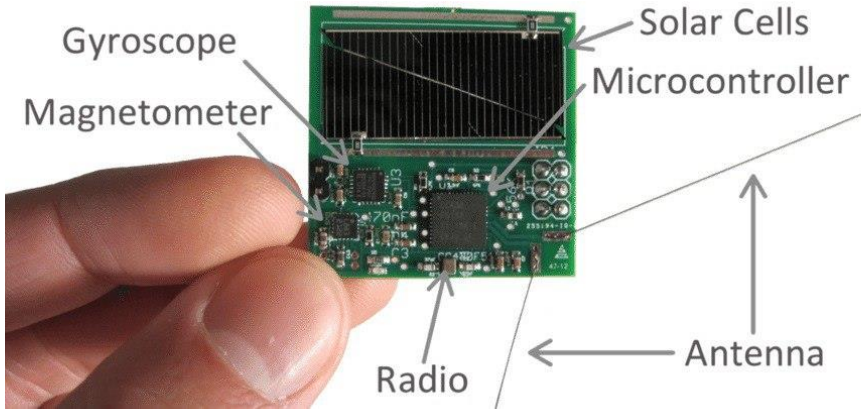

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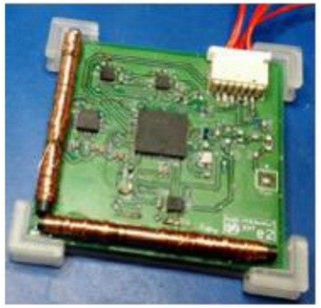

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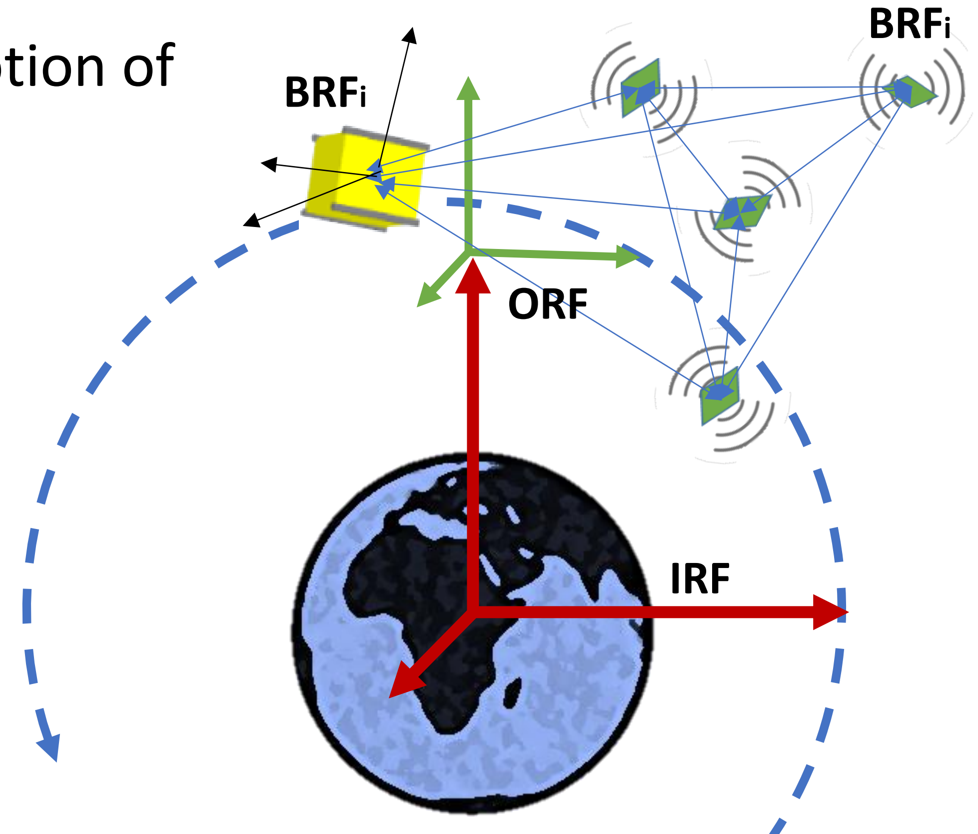
Problem statement

- Considered:
  - Chief satellites with known orbital and attitude motion at near circular orbit
  - Deputy femtosatellites deployed from chief satellites
  - Antennas of the satellites in the swarm are isotropic or half-wave antennas
  - Deputy femtosatellites attitude is considered known

- It is necessary to estimate relative translational motion of femtosatellites swarm relative to chief satellites

- Reference frames:
  - Inertial (IRF)
  - Orbital (ORF)
  - Body (BRF)



Motion system and measurement system

- Motion equations:

Orbital motion in ORF

Aerodynamic drag

Attitude motion in IRF

$$
\begin{cases}
\dot{\mathbf{r}} = \boldsymbol{\upsilon}, \\
\dot{\upsilon}_x = -2\omega_0 \upsilon_z - \dfrac{\rho C_d}{2m} S(\upsilon_{orb} + \upsilon_x)^2 (\lambda_x \lambda_y - \lambda_0 \lambda_z), \\
\dot{\upsilon}_y = -\omega_0^2 y, \\
\dot{\upsilon}_z = 3\omega_0^2 z + 2\omega_0 \upsilon_x, \\
\dot{\boldsymbol{\lambda}} = \dfrac{1}{2} \boldsymbol{\lambda} \circ \boldsymbol{\omega}, \\
\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} = \mathbf{M}_{ext}.
\end{cases}
$$

$\boldsymbol{\lambda} = [\lambda_0, \lambda_x, \lambda_y, \lambda_z]$: Attitude quaternion

ω_0 : Angular velocity of ORF center in IRF

υ_{orb} : Translational velocity of ORF center in IRF

$\mathbf{I}$: Satellite inertia tensor

$\mathbf{M}_{ext}$: External torque

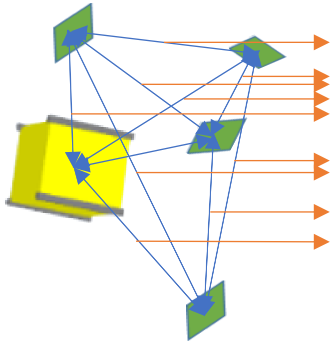
- State vector of N femtosatellites to find:

$$\mathbf{x} = [\mathbf{r}_1^T, \boldsymbol{\upsilon}_1^T, \ldots \mathbf{r}_N^T, \boldsymbol{\upsilon}_N^T]^T$$

- Measurements vector of *M+N(N-1)/2* RSSI (received signal strength indicator) between *M* chief and *N* deputy:

$$\mathbf{y} = [y_{c_1-d_1}, \ldots y_{c_M-d_N}, y_{d_1-d_2}, \ldots y_{d_{N-1}-d_N}]$$

- Model of antenna signal



$\mathbf{y}(t)$

$$\mathrm{P}_R = \mathrm{P}_T G_R G_T \frac{\lambda^2}{4\pi r^2}, \quad \tilde{r}^2 = \frac{4\pi \mathrm{P}_R}{\lambda^2 \mathrm{P}_T} = \frac{r^2}{G_R G_T},$$

Measurement model:

$$y = \tilde{r} + \delta r = \frac{r}{\sqrt{G_R G_T}} + \delta r,$$

$G \equiv 1$, Isotropic antenna

$$G = \frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta},$$ Half-wave antenna

Silver S. «Microwave Antenna Theory and Design», McGraw Hill Book Company. 1984.

- Sufficient observability criteria: the system $\begin{cases}\dot{\mathbf{x}} = \mathbf{f}(t,\mathbf{x}), \\ \mathbf{y} = \mathbf{h}(t,\mathbf{x})\end{cases}$ is observable on $[t_0, t_1]$, if:

1. $km = n$, where n: Length of state vector
   m: Length of measurement vector
   k: Number of existing derivatives

2. Mapping of observability **H** is differentiable:

$$\mathbf{H} = \begin{bmatrix} \mathbf{y} \\ \vdots \\ \dfrac{d^{k-1}\mathbf{y}}{(dt)^{k-1}} \end{bmatrix} \quad \mathbf{J} = \frac{\partial \mathbf{H}}{\partial \mathbf{x}_0} = \begin{bmatrix} \dfrac{\partial y}{\partial x_0^1} & \cdots & \dfrac{\partial y}{\partial x_0^n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial y^{(k-1)}}{\partial x_0^1} & \cdots & \dfrac{\partial y^{(k-1)}}{\partial x_0^n} \end{bmatrix}$$

The system is locally observable over the interval [0,T] if the mapping from the initial state x_0 to output profile y(t) (t∈[0,T]) is local bijection (i.e., if the initial state can be reconstructed by the outputs on this interval)

Bartosiewicz, Z., "Local Observability of Nonlinear Systems," Systems & Control Letters, Vol. 25, No. 4, 1995, pp. 295–298. doi:10.1016/0167-6911(94)00074-6

3. The Jacobi matrix **J** satisfies the uniform ratio of the major minors:

$$\exists \varepsilon > 0 \quad \rightarrow \quad |\Delta_1| \geq \varepsilon, \quad \frac{|\Delta_2|}{|\Delta_1|} \geq \varepsilon, \quad \ldots \quad \frac{|\Delta_n|}{|\Delta_{n-1}|} \geq \varepsilon \quad \forall x \in E^n$$

Kou S.R. "Observability of Nonlinear Systems". Information and Control, 1973, Vol. 22, pp. 89-99.

Numerical observability of the system

Andrew J. Whalen "Observability and Controllability of Nonlinear Networks: The Role of Symmetry"

- Criteria of observability: the system $\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) \\ \mathbf{y} = \mathbf{h}(t, \mathbf{x}) \end{cases}$

  is observable if and only if $\exists k \Rightarrow$ singular numbers are equal in modulus :

$$g(\mathbf{x}) = 1, \quad \text{where} \quad g(\mathbf{x}) = \frac{\left|\sigma_{\min}(\mathbf{J}^T \mathbf{J})\right|}{\left|\sigma_{\max}(\mathbf{J}^T \mathbf{J})\right|}$$

$\sigma -$ singular numbers

- If $g(\mathbf{x}) < 1$ it cannot be stated that system does not local observable
- Some empirical criteria can be used. For example, can be introduced a threshold P to separate the observed subspace in state vector space:

$$\sigma_1, \ldots \sigma_j > P, \qquad \sigma_{j+1}, \ldots \sigma_n \leq P$$

For observed subspace — For unobserved subspace

Extended Kalman filter application

- Linearized dynamic model
- Linearized measurement model

$$\begin{cases} \mathbf{x}_k^- = \Phi_k \mathbf{x}_{k-1}, \\ \mathbf{y}_k = H\mathbf{x}_k + \boldsymbol{\upsilon}_k. \end{cases}$$

$$\tilde{\mathbf{y}} = \frac{\left|\mathbf{r}_{deputy} - \mathbf{r}_{chief}\right|}{\sqrt{G_R G_T}}$$

- Prediction step

$$\mathbf{x}_k^- = \Phi_k \mathbf{x}_{k-1}$$

$$P_k^- = \Phi_k P_{k-1} \Phi_k^T + Q^\sim$$

$$Q^\sim = \int \Phi_k D Q D^T \Phi_k^T dt$$

- Update step

$$K = P_k^- H_k^T (H_k P_k^- H_k^T + R)$$

$$\mathbf{x}_k = \mathbf{x}_{k-1} + K(\mathbf{y}_k^{real} - \mathbf{y}_k^{model})$$

$$P_k = (E - KH_k) P_k^-$$

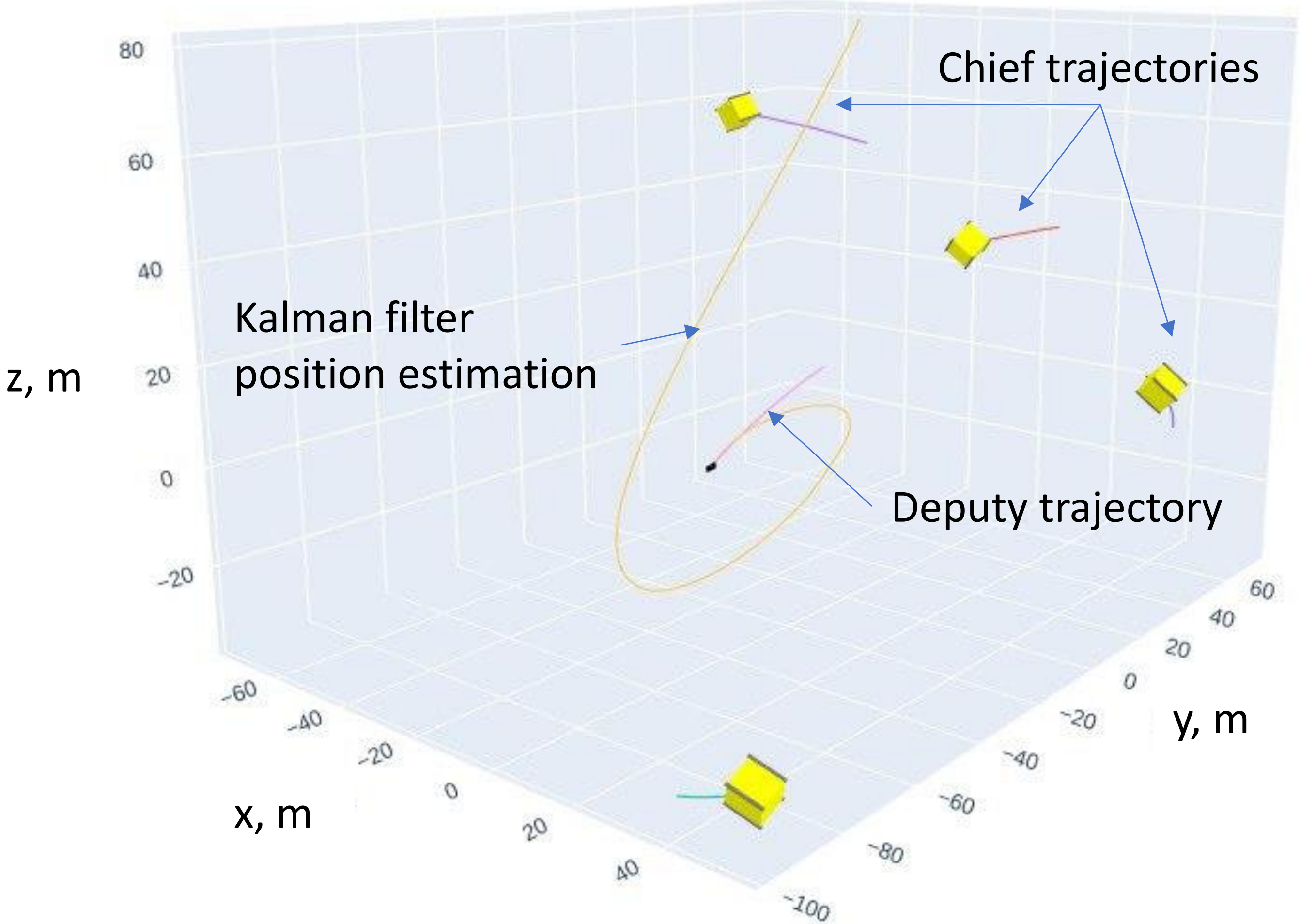
$$\Phi = \begin{bmatrix} \Phi_1 & \cdots & 0_{12\times12} \\ \cdots & \ddots & \cdots \\ 0_{12\times12} & \cdots & \Phi_n \end{bmatrix}$$

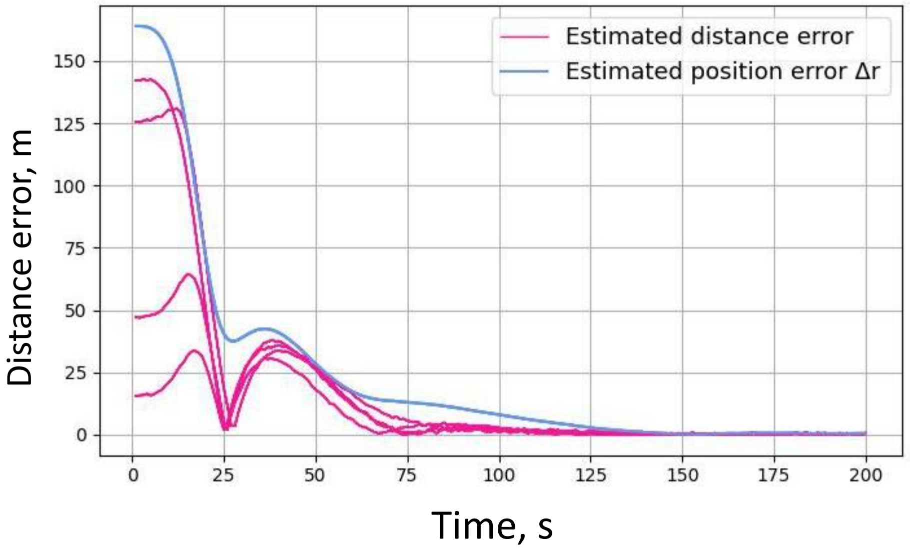
$$P(t=0) = \begin{bmatrix} P_1 & \cdots & 0_{12\times12} \\ \cdots & \ddots & \cdots \\ 0_{12\times12} & \cdots & P_1 \end{bmatrix}$$

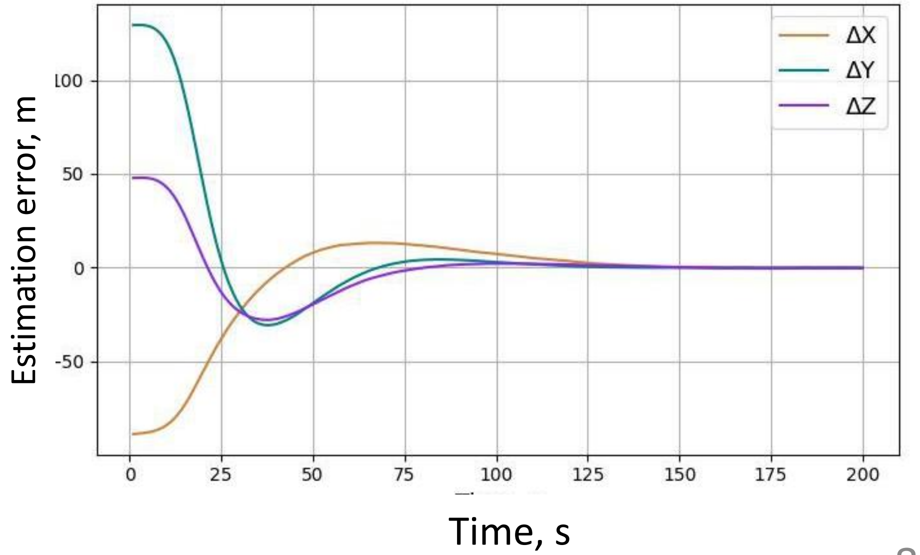
$$Q = \begin{bmatrix} Q_1 & \cdots & 0_{6\times6} \\ \cdots & \ddots & \cdots \\ 0_{6\times6} & \cdots & Q_1 \end{bmatrix}$$

Example of the system with full observability

- 4 chief spacecrafts and 1 deputy femtosatellite
- Isotropic antennas $G \equiv 1$
- State vector $\mathbf{x} = [\mathbf{r}_1^T, \mathbf{v}_1^T]^T$

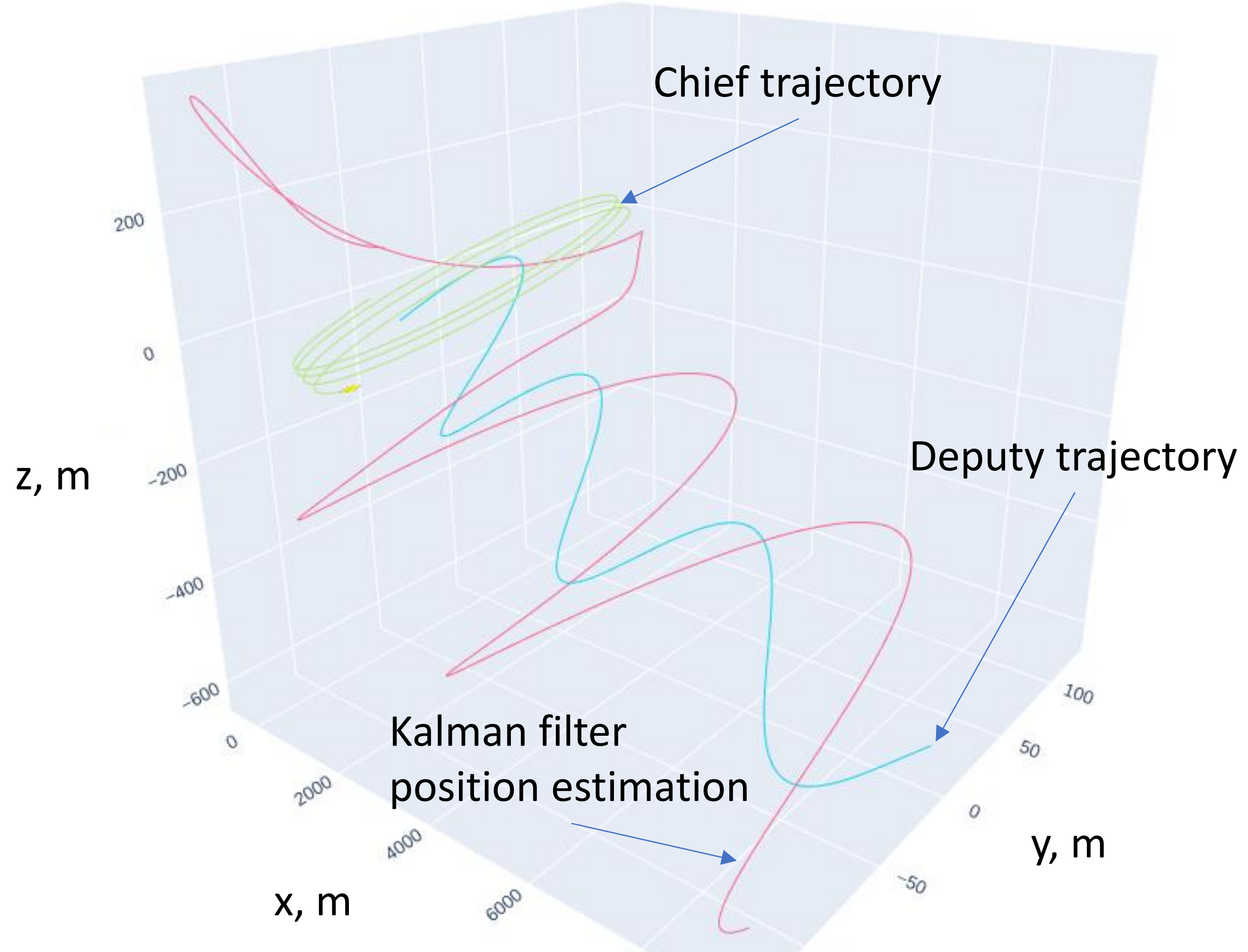


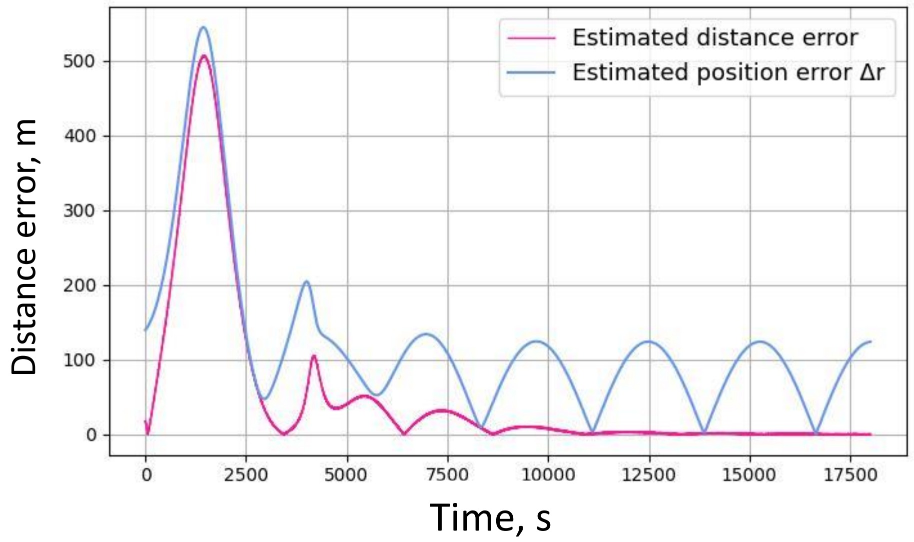


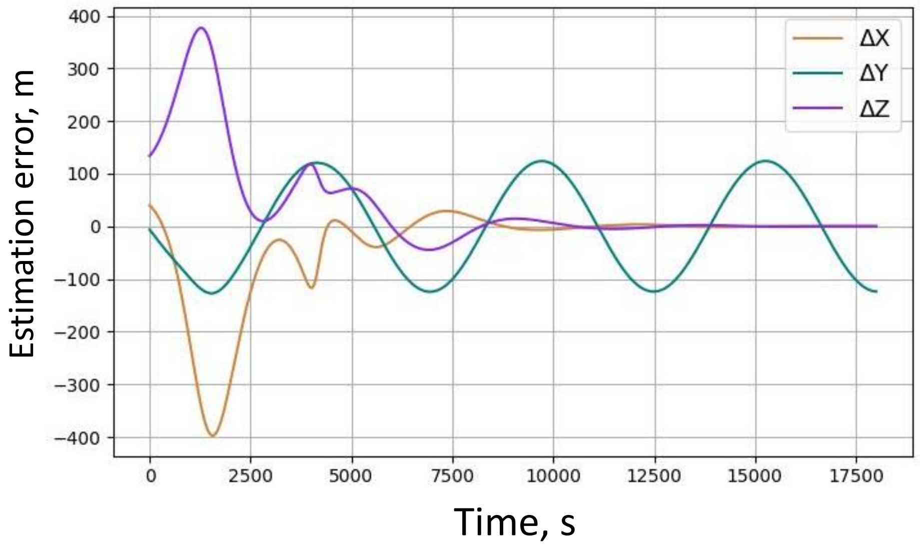


Example of the system with local observability

- 1 chief and 1 deputy
- Isotropic antennas $G \equiv 1$
- State vector $\mathbf{x} = [\mathbf{r}_1^T, \boldsymbol{\upsilon}_1^T]^T$

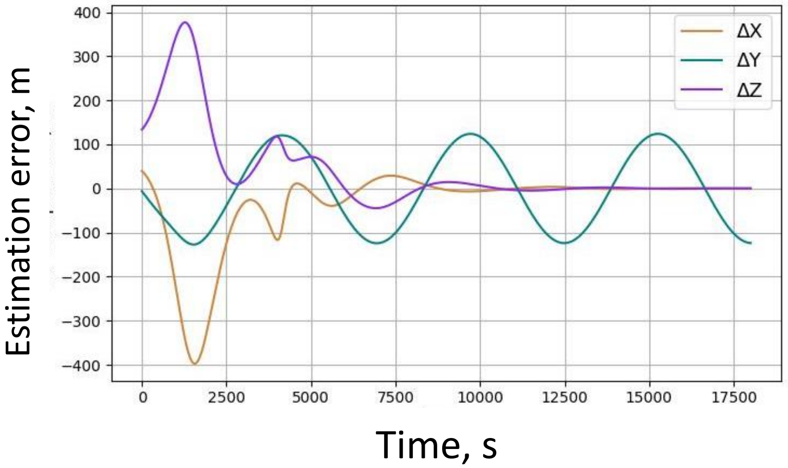






Local observability of the system

- 1 chief and 1 deputy



$$\mathbf{x} = [x, y, z, \upsilon_x, \upsilon_y, \upsilon_z] \quad \rightarrow \quad \mathbf{x} = [x, z, \upsilon_x, \upsilon_z]$$

Singular values: $\boldsymbol{\sigma} \approx [1, 1, 10^{-2}, 10^{-12}]$

The uniform ratio of the minors is not fulfilled (there is no sufficient observability condition)

Numerical modeling

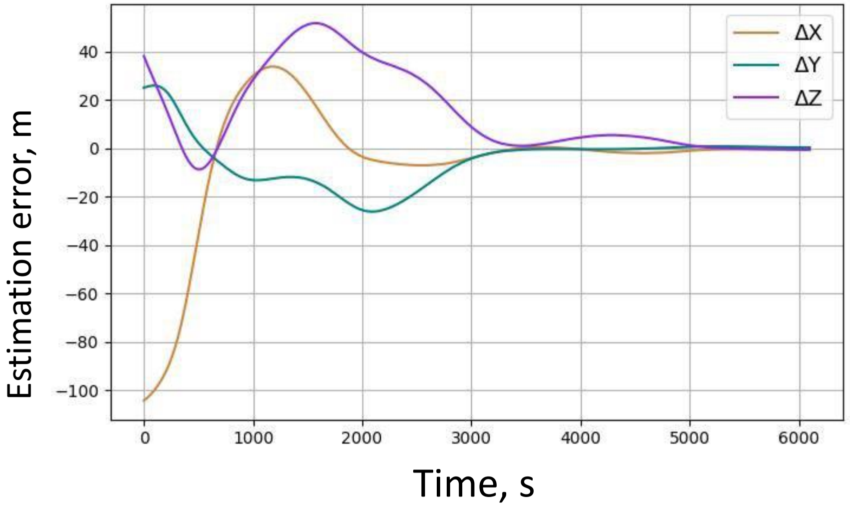
System is unobservable

The uniform ratio of the minors is fulfilled

$$\frac{|\Delta_i|}{|\Delta_{i-1}|} \geq 10^{-6} \sim 10^{-5}$$

System is locally observable

- 2 chiefs and 1 deputy



State vector: $\mathbf{x} = [x, y, z, \upsilon_x, \upsilon_y, \upsilon_z]$

Singular values: $\boldsymbol{\sigma} \approx [1, 1, 10^{-1}, 10^{-1}, 10^{-3}, 10^{-6}]$

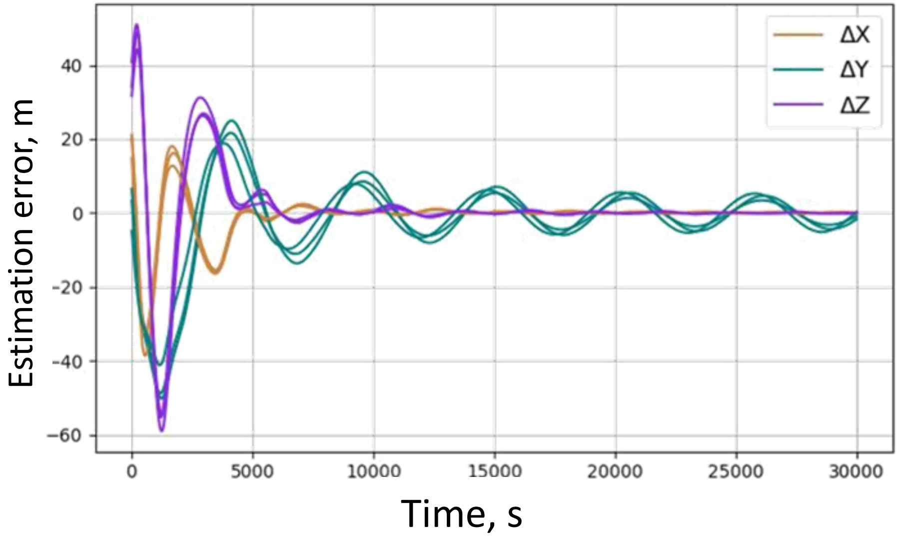
The uniform ratio of the minors is fulfilled

System is fully observable

$$\frac{|\Delta_i|}{|\Delta_{i-1}|} \geq 10^{-5} \sim 10^{-4}$$

Local observability depending on chiefs relative motion

- 2 chiefs satellites and 3 deputies
- Relative distance between the chiefs is 50-100m



State vector:

$$\mathbf{x} = [x, y, z, \upsilon_x, \upsilon_y, \upsilon_z]$$

Singular values:

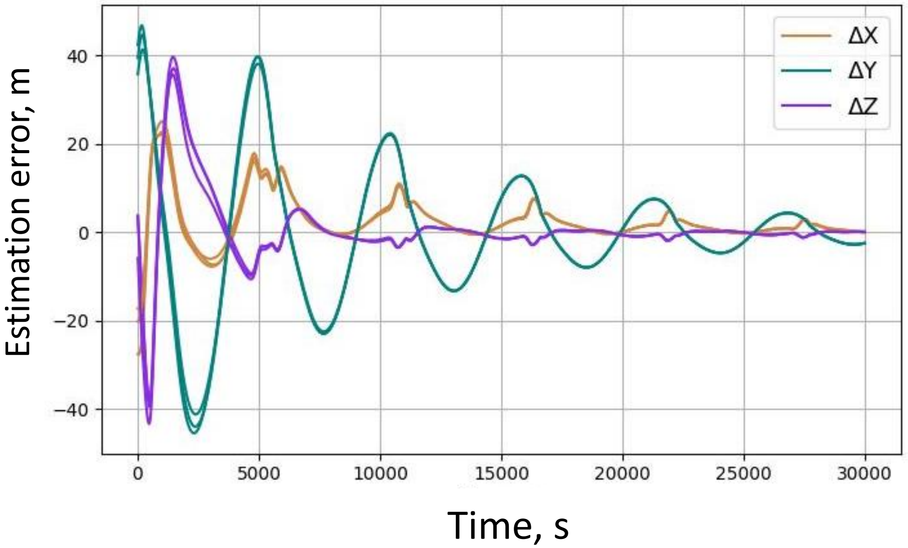
$$\boldsymbol{\sigma} \approx [1, 1, 10^{-1}, 10^{-1}, 10^{-6}, 10^{-12}]$$

The uniform ratio of the minors is not fulfilled

$$\frac{|\Delta_4|}{|\Delta_5|} < 10^{-12}$$

System is locally observable

- Relative distance between the chiefs is 200-400m



State vector: $\mathbf{x} = [x, y, z, \upsilon_x, \upsilon_y, \upsilon_z]$

Singular values: $\boldsymbol{\sigma} \approx [1, 1, 10^{-1}, 10^{-1}, 10^{-3}, 10^{-6}]$

The uniform ratio of the minors is fulfilled

$$\frac{|\Delta_i|}{|\Delta_{i-1}|} \geq 10^{-5} \sim 10^{-4}$$

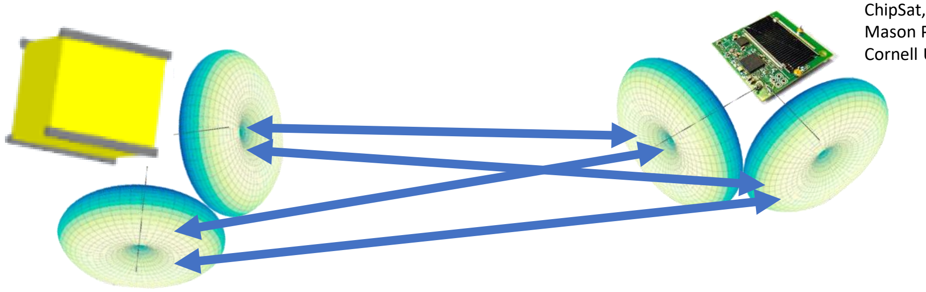
System is fully observable

Next steps: case of anisotropic antennas

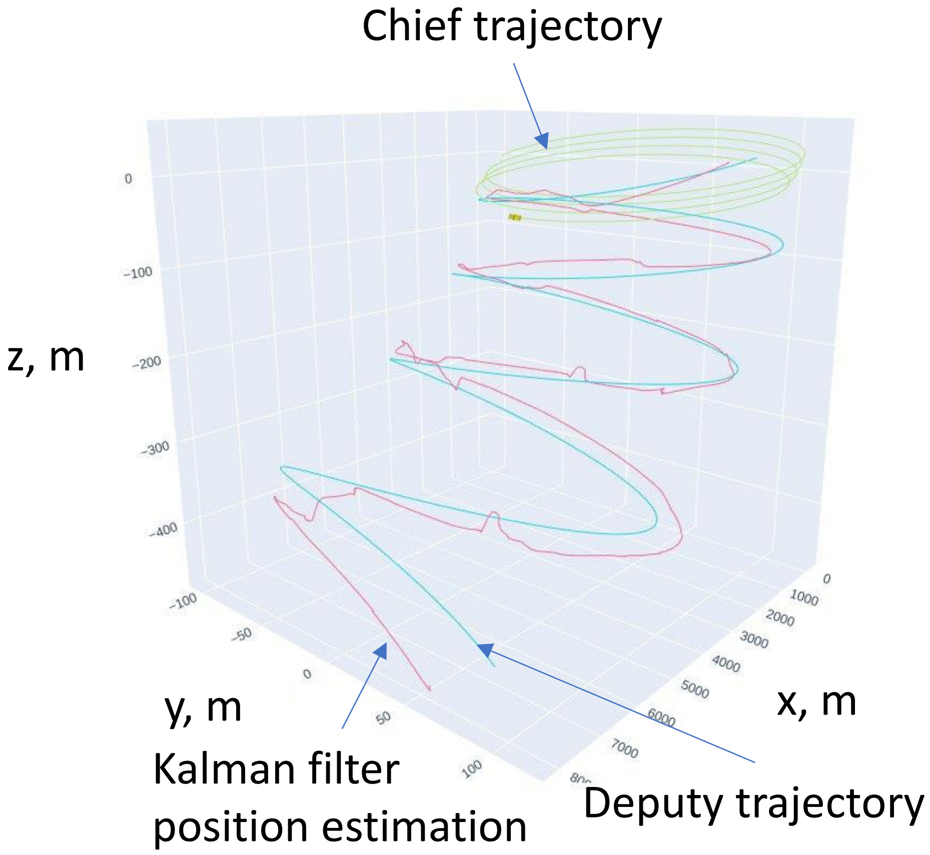
- Relative motion observability including the deputies attitude motion using case of anisotropic antennas is currently under the investigation
- 1 chief spacecrafts and 1 deputy femtosatellite
- Half-wave dipole antennas model $G = \dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}$
- State vector $\mathbf{x} = [\mathbf{r}_1^T, \boldsymbol{\upsilon}_1^T, \boldsymbol{\lambda}_1^T, \boldsymbol{\omega}_1^T]^T$
- System matrix and measurement matrix:

$$\Phi_1 = \begin{bmatrix} E_{3\times3} & 0_{3\times3} & E_{3\times3}\cdot dt & 0_{3\times3} \\ 0_{3\times3} & E_{3\times3} & 0_{3\times3} & E_{3\times3}\cdot\dfrac{dt}{2} \\ A & 0_{3\times3} & B & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & C \end{bmatrix}$$

$$H = \frac{\partial \mathbf{z}}{\partial \mathbf{x}} = \begin{bmatrix} H_{1,1}^{cd} & H_{1,2}^{cd} & \cdots & \cdots & H_{1,N}^{cd} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ H_{M,1}^{cd} & H_{M,2}^{cd} & \cdots & \cdots & H_{M,N}^{cd} \\ H_{1,2}^{dd} & H_{2,1}^{dd} & 0_{N\times L_z} & \cdots & 0_{N\times L_z} \\ H_{1,3}^{dd} & 0_{N\times L_z} & H_{3,1}^{dd} & \cdots & 0_{N\times L_z} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0_{N\times L_z} & 0_{N\times L_z} & 0_{N\times L_z} & \cdots & H_{N-1,N}^{dd} \end{bmatrix}$$



ChipSat,
Mason Peck lab,
Cornell University



Conclusions

- Possibility of relative motion estimation using measurements of communication signal magnitude is studied in this paper
- It is shown that for the case of isotropic antennas the system with one chief satellite is locally observable, and in case of two chiefs satellites the system is fully observable
- Depending on relative distance of two chiefs the uniform ratio of the minors can be not fulfilled and the system can be not observable
- As continuation of this study the case of measurements of anisotropic antennas signal magnitude is under the investigation

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https://rscf.ru/project/24-11-00038/

Thank you for your attention!