Reference orbit construction for tetrahedral formation on elliptical orbit

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Motivation

1) Slava G. Turyshev, Sheng-wey Chiow, and Nan Yu
Searching for new physics in the solar system with tetrahedral spacecraft formations
Phys. Rev. D, 109:084059, Apr 2024
doi: 10.1103/PhysRevD.109.084059

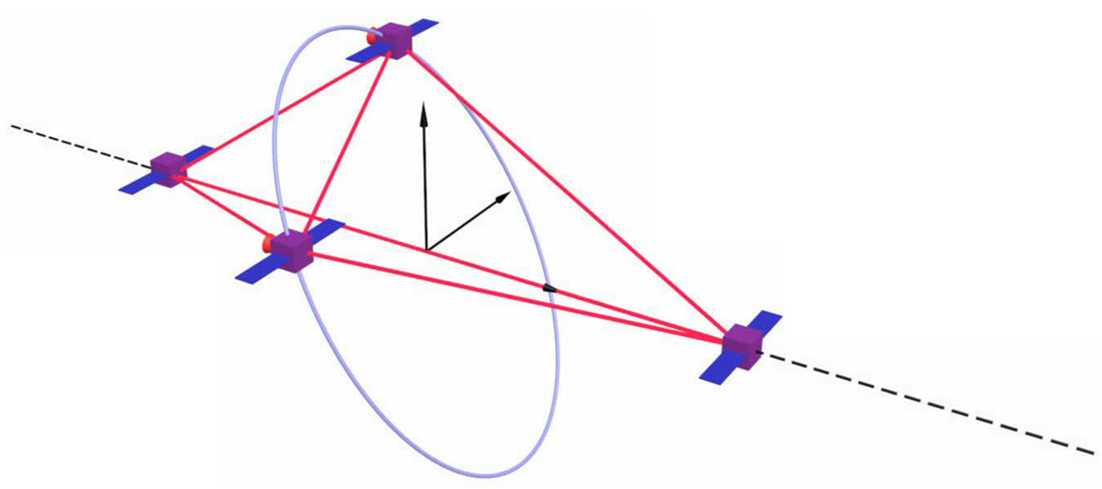
2) S.A. Bogachev, M.Yu. Ovchinnikov, V.A. Shuvalov
Main Tasks and Possibilities to Study the Magnetic Field and Electric Currents in the Earth's Magnetosphere by Small Spacecraft
Cosmonautics and rocket science, 2024, 1(134), pp. 158-169 (in russian)

Need for a tetrahedral formation

1) to study gravity gradient tensor
e=0.6, a=1AU, r=1000km

2) to study Earth's magnetosphere
e=0.7, a=20R_{Earth}, r=100km

So, distance between satellites to orbit semi-axis ratio is a small parameter



Tschauner–Hempel linearized equations

- elliptic reference orbit
- in orbital frame

$$x'' - 2y' - \frac{3x}{1+e\cos\nu} = 0$$
$$y'' + 2x' = 0$$
$$z'' + z = 0$$

- bounded solution for satellite number j has the form

$$x_j(\nu) = A_j(1+e\cos\nu)\sin\nu + B_j(1+e\cos\nu)\cos\nu$$
$$y_j(\nu) = A_j(2+e\cos\nu)\cos\nu - B_j(2+e\cos\nu)\sin\nu + C_j$$
$$z_j(\nu) = D_j\sin\nu + E_j\cos\nu$$

Need to maintain constant tetrahedral volume

to make accurate measurements of gravitational/electromagnetic field

We did such a thing for circular orbits:
S. Shestakov, M. Ovchinnikov, Y. Mashtakov
Analytical Approach to Construction of Tetrahedral Satellite Formation
Journal of Guidance, Control and Dynamics, 2019, V. 42, No. 12, pp. 2600-2614
doi: 10.2514/1.G003913

Every satellite with j=1,2,3 has its own A_j, B_j, C_j, D_j, E_j
Combine constants into vectors $\mathbf{A} = (A_1, A_2, A_3)^T$

Then for the volume we have

$$\mathbb{V} = \frac{p^3}{6(1+e\cos\nu)^3}\det\|\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\|$$

We calculate the volume, here angle brackets are mixed product of vectors

$$\det\|\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3\| = (1+e\cos\nu)\cdot\Big(\frac{1}{2}\cdot(\langle\mathbf{A},\mathbf{C},\mathbf{D}\rangle + \langle\mathbf{B},\mathbf{C},\mathbf{E}\rangle - e\langle\mathbf{A},\mathbf{B},\mathbf{E}\rangle) + $$
$$- 2\cos\nu\cdot\langle\mathbf{A},\mathbf{B},\mathbf{E}\rangle +$$
$$- 2\sin\nu\cdot\langle\mathbf{A},\mathbf{B},\mathbf{D}\rangle +$$
$$\frac{1}{2}\cos 2\nu\cdot(-\langle\mathbf{A},\mathbf{C},\mathbf{D}\rangle + \langle\mathbf{B},\mathbf{C},\mathbf{E}\rangle - e\langle\mathbf{A},\mathbf{B},\mathbf{E}\rangle) +$$
$$\frac{1}{2}\sin 2\nu\cdot(\langle\mathbf{A},\mathbf{C},\mathbf{E}\rangle + \langle\mathbf{B},\mathbf{C},\mathbf{D}\rangle - e\langle\mathbf{A},\mathbf{B},\mathbf{D}\rangle)\Big)$$

We decompose a numerator and a denominator into trigonometric polynomial of degrees of true anomaly

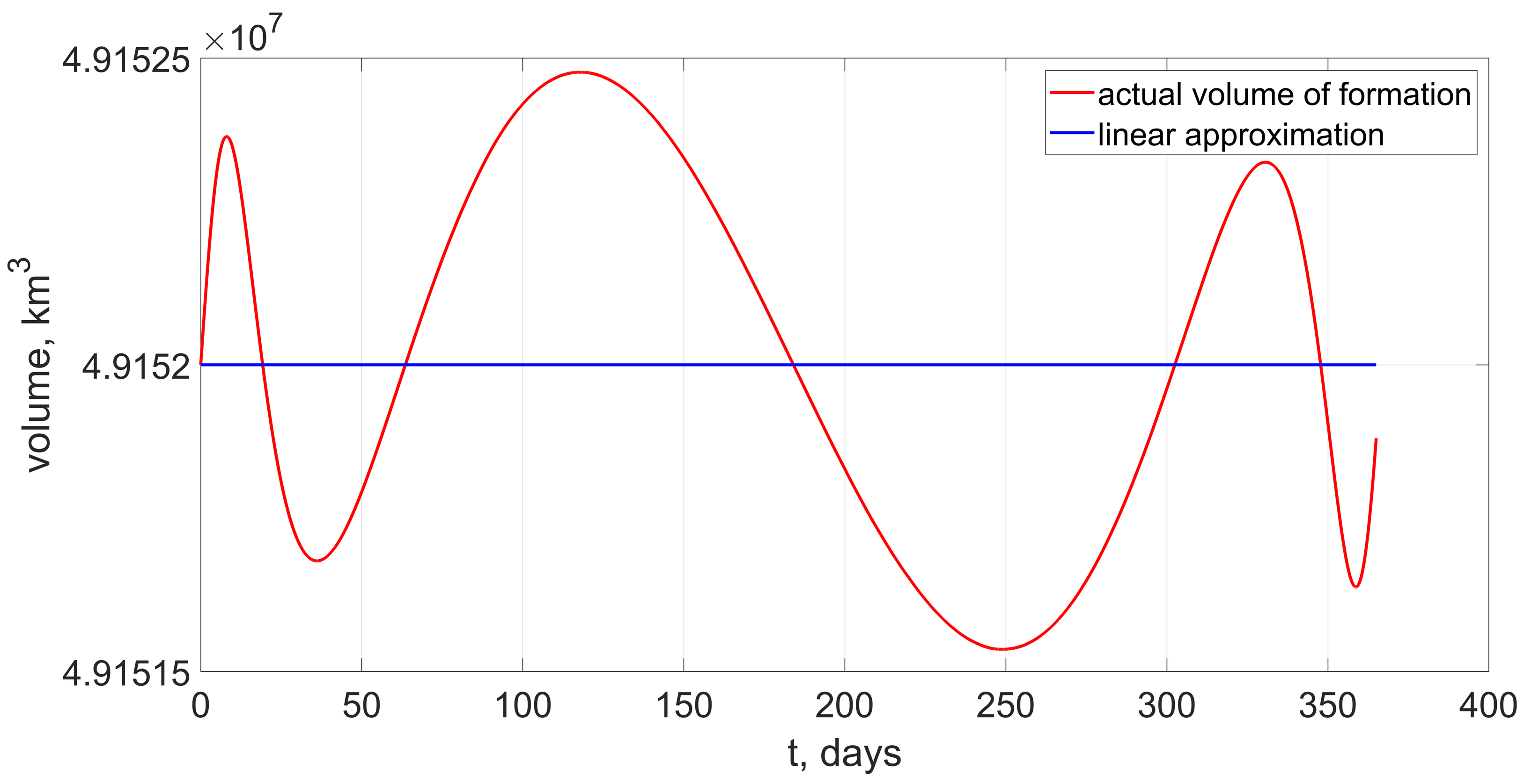
We get the necessary and sufficient conditions for the volume to conserve, k is arbitrary here

$$\langle\mathbf{A},\mathbf{B},\mathbf{D}\rangle = 0$$
$$\langle\mathbf{A},\mathbf{C},\mathbf{E}\rangle + \langle\mathbf{B},\mathbf{C},\mathbf{D}\rangle = 0$$
$$\langle\mathbf{A},\mathbf{B},\mathbf{E}\rangle = -ke$$
$$\langle\mathbf{A},\mathbf{C},\mathbf{D}\rangle = k$$
$$\langle\mathbf{B},\mathbf{C},\mathbf{E}\rangle = k$$

Numerical experiment

Motion of tetrahedral formation around the Sun
a=1AU, e=0.6, central gravity
initial conditions are such that volume is conserved in Tschauner-Hempel model

In nonlinear model the volume is still constant



Not only volume must be maintained

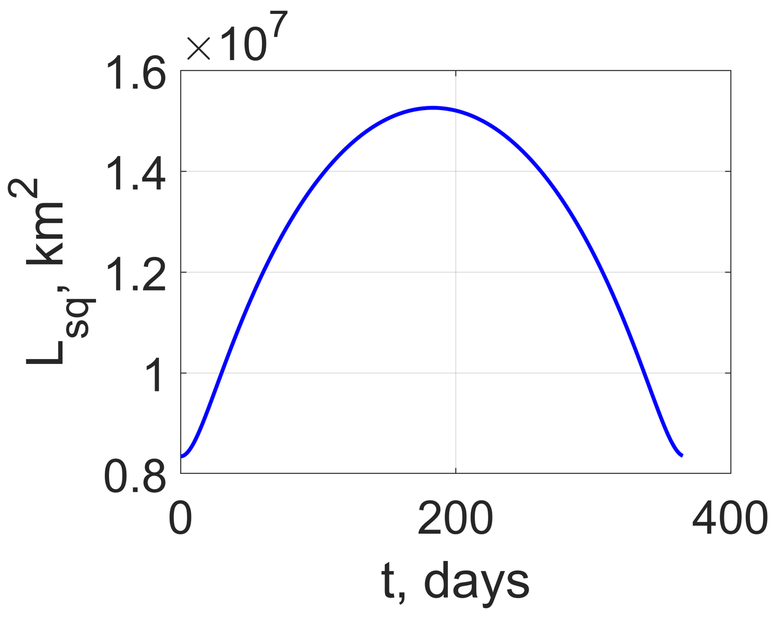
Formation shape is also of interest in applications
e.g. for intersatellite links to work
distances and relative angles must be under control

But shape is not a single number
We choose quality parameter $\mathbb{Q} \sim \mathbb{V}^{2/3}\cdot\mathbb{L}_{sq}^{-1}$

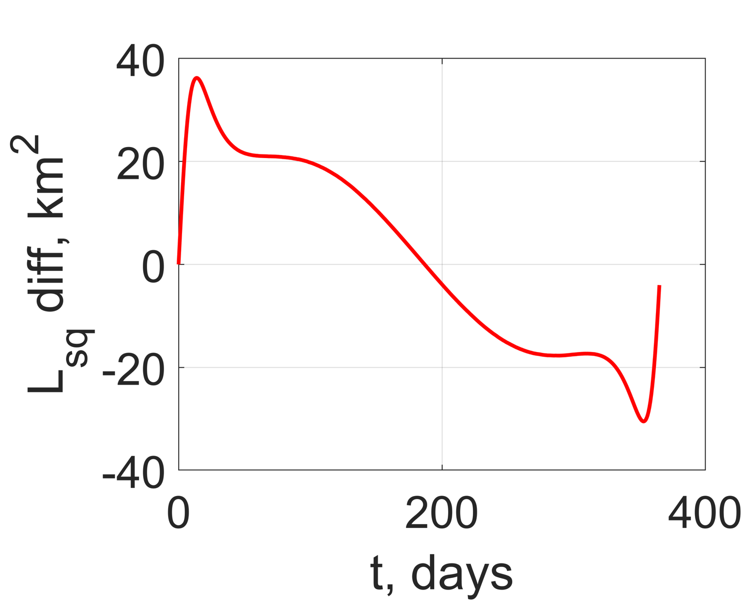
Here L_{sq} is sum of squares of lengths of all edges

With constant volume it is impossible to maintain constant quality unless the reference orbit is circular

But it is possible to analytically minimize the variations in quality



L_{sq} numerically obtained



Difference between numerical and theoretical values of L_{sq}